Nonlinear MHD stability of rotating plasmas in a mirror/θ–pinch geometry

Ahmet Y. Aydemir

Institute for Fusion Studies
The University of Texas at Austin
Abstract

In a previous work, we studied the magnetohydrodynamic (MHD) equilibrium and stability of a mirror plasma in which an externally applied radial electric field drives a strong azimuthal rotation[1]. Although the interchange-stabilization through flow-shear[2] was confirmed, centrifugally confined "detached states" obtained in this geometry were found to be linearly unstable to a wide range of other fluid modes driven by the rotation itself. These negative linear stability results left open the question of whether these modes would be nonlinearly stabilized at modest amplitudes or have catastrophic consequences. The goal of this present work is to follow the nonlinear evolution of some of these modes and determine their effect on confinement in this geometry.

Geometry and Equations

- A straight cylinder, with a perfectly conducting wall and periodically identified ends.

- Azimuthal (poloidal) symmetry.

- \( z = R \zeta \), with \( R = 1 \).

- Minor radius \( a = 1 \).
Single fluid resistive MHD equations

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \]

\[ \rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = \mathbf{J} \times \mathbf{B} - \nabla p, \]

\[ \frac{\partial \mathbf{A}}{\partial t} = \mathbf{u} \times \mathbf{B} - \eta \mathbf{J}, \]

\[ \frac{\partial p}{\partial t} + (\mathbf{u} \cdot \nabla) p + \gamma p \nabla \cdot \mathbf{u} = \kappa_\perp \nabla^2 p + \kappa_\parallel \nabla^2_- (p/\rho), \]

\[ \mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{J} = \nabla \times \mathbf{B}. \]
Equilibria with rotation

• Starting with a static equilibrium, we generate different families of rotating equilibria by specifying the rotation frequency profile, $\Omega(\chi) = \Omega_0(\chi)f(\chi)$ and incrementally increasing $\Omega_0$

• Profiles vary from nearly rigid-body rotation (low shear) to those with very high shear.

• Frequencies are pushed close to the radial equilibrium limit where the plasma starts to “fly apart”.

• As expected, the equilibria have $T = T(\chi)$, due to rapid parallel thermal conduction, but finite density (and pressure) gradients within the flux surfaces.
Typical rotation profiles

Various profiles:

- For $a$, $b$, and $c$:
  \[
  \Omega(\chi) = \Omega_0 \left(1 - \exp\left(-\frac{(1 - x)^2}{w^2}\right)\right) \div \left(1 - \exp\left(-\frac{1}{w^2}\right)\right), \text{ where } x = \frac{\chi}{\chi_{\text{max}}}. 
  \]
  a) $w = 0.273$, b) $w = 0.546$, c) $w = 1.09$.

- For $d$:
  \[
  \Omega(\chi) = \Omega_0 \left(1 - x^2\right)^2, \text{ with } \lambda = 0.6. 
  \]
An equilibrium with medium shear for $\Omega_0=0.73$ [Profile (b)]
Evolution of a high-shear equilibrium with increasing frequency [Profile (d)]
• As expected, the mirror equilibrium without rotation is strongly unstable to interchange (flute) modes.

• All m’s are expected to be unstable.

• Viscous and other dissipative terms in the system help “damp” high mode numbers.

• We typically examine modes with m=1,2,3 … 9.
The interchange modes that exist for $\Omega_0=0$ are never stabilized.

This “rigid-rotation” profile is clearly very destabilizing, as expected.

“Shear stabilization” is more effective for higher mode numbers and occurs very early, long before centrifugal forces become relevant.

The real frequency scales linearly with the rotation frequency: $\omega_r \sim m \Omega$.

There is a competing mode with $\omega_r \sim 0$!
With “shear stabilization”, some of the modes are replaced by a zero-frequency mode for $0.1 < \Omega_0 < 0.3$.

These new modes, possibly due to a “MRI/Parker instability”, have only a very weak dependence on rotation frequency.

For $\Omega_0 > 0.3$, the interchange modes reappear.
Eigenfunction differences between the “zero-frequency” and interchange modes.

- The “zero-frequency” mode typically has $\omega_r/\gamma \sim 10^{-2}$, and $\omega_r/\Omega_0 \sim 10^{-3}$.
- Unlike the interchange mode, it has finite $k_\parallel$.
- It is “localized” to the edge.
- For the mode shown ($m = 8$), we have $\omega_1 = (4.95 \times 10^{-2}, \sim 0)$, and $\omega_2 = (8.60 \times 10^{-2}, 2.19)$ at $\Omega_0 = 0.3$
- The simultaneous presence of these two (?) modes sometimes makes it difficult to converge to an eigenvalue.
Eigenfunction differences between the “zero-frequency” and interchange modes.

- The “zero-frequency” mode is due to a “MRI/Parker instability.”

- It is a current driven mode with a finite parallel mode number.

- It can be understood in terms of an “attraction” between current filaments on a sinusoidally perturbed surface.
The interchange mode is quickly "shear-stabilized."

The "zero-frequency" mode persists for all $\Omega$.

The small differences in the growth rate for different $m$'s (perpendicular mode no.) is probably due to dissipation.
Straight theta-pinch stability (no mirror fields)

• In order to better understand some of the rotating mirror stability results, here we remove the mirror fields and look at a straight theta-pinch with rotation.
• The problem is simplified by the additional symmetry.
• Since the mirror field itself has a destabilizing influence the results in its absence will tend to overestimate stability (or underestimate the linear growth rates) for the observed modes.

Rotation frequency profiles will be parametrized as:

- $\Omega(x) = \Omega_0 \left(1 - x^\lambda\right)^2$, or
- $\Omega(x) = C\Omega_0 x^{\lambda_1} (1 - x)^{\lambda_2}$, where $x \equiv \chi/\chi_{max}$, and $\chi$ is the toroidal flux function.

We will vary plasma $\beta$, and also look at the effects of an inverted density profile.
Straight $\theta$–pinch growth rates for: $\lambda = 2 \quad \beta = 10\%$

- $\Omega = 0.10$
- $\Omega = 0.3$
- $\Omega = 0.5$
Straight $\theta$ –pinch growth rates for $\lambda = 0.6 \quad \beta = 10\%$
Straight $\theta$-pinch growth rates for: $\lambda = 2$, $\beta = 1\%$

$\Omega(\Omega = 0.025)$

$\Omega(\Omega = 0.1)$

$\Omega(\Omega = 0.15)$

$\Omega(\Omega = 0.25)$
Straight $\theta$–pinch: eigenfunctions for $m/n = 5/5$
Straight $\theta$–pinch growth rates for: $\lambda = 0.6 \quad \beta = 1\%$

2/15/2006

ICC 2006, Austin, TX
Straight $\theta$–pinch growth rates for: $\Omega(0) = \Omega(1) = 0$

\begin{align*}
\Omega &= 0.20 \\
\Omega &= 0.3 \\
\Omega &= 0.4 \\
\Omega &= 0.475
\end{align*}
Straight $\theta$–pinch growth rates for: $\frac{\partial \rho}{\partial r} > 0$.

- Examine the effects of an “inverted” density profile.
Straight $\theta$–pinch growth rates for: $\frac{\partial \rho}{\partial r} > 0$. 

\begin{align*}
\Omega &= 0.2 \\
\Omega &= 0.3 \\
\Omega &= 0.4 \\
\Omega &= 0.475
\end{align*}
Nonlinear stability in $\theta$-pinch geometry

- Since it is difficult to separate various unstable modes in the mirror geometry, we initially study nonlinear stability in a $\theta$-pinch.

- We use a rotation profile with high shear: $\Omega = \Omega_0 (1 - \rho)$, where $\rho$ is flux surface label.

- Our preliminary results are presented on the following pages.
Linear stability for $\Omega = \Omega_0 (1 - \rho)$

Growth rates for $\Omega_0 = 1.8$

$\gamma_{\text{max}} = 3.597 \times 10^{-1}$
Time evolution of density profile, with $\Omega=1.8$, and assumed symmetry in z direction.
Linear stability for $\Omega = \Omega_0 (1 - \rho)$

Growth rates for $\Omega_0 = 2.6$

$\gamma_{\text{max}} = 7.735 \times 10^{-1}$
Axial Field profile evolution for $\Omega=2.6$
Summary

• In our earlier work, we had shown that there are a large number of unstable modes in the rotating mirror geometry, driven by the rotation itself.

• Our preliminary results in a simplified $\theta$–pinch indicate that both
  
  – Interchange-like modes with $k_\parallel = 0$, and
  
  – Parker/MRI (?) modes with finite $k_\parallel$

  can grow to large amplitudes, possibly leading to complete loss of confinement.

• In our calculations, we see a tendency towards “wall-confined” plasmas at high-rotation rates.