Improved Stability & Confinement in a High-\(\beta\) Relaxed Plasma State

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Introduction

- **Taylor relaxed state – force free ($\beta = 0$)**
  - Minimum magnetic energy: $W_m = \int B^2/2\mu_0 \, d\tau$
  - Conserving magnetic helicity: $K = \int A \cdot B \, d\tau$
  $\Rightarrow$ “Beltrami condition”: $\nabla \times B = \lambda B$, $\lambda$ is a global constant

- **Modern relaxation principle /two fluid theory (finite $\beta$)**
  - Minimizing total magnetofluid energy: $W_{mf} = \int (B^2/2\mu_0 + mn u^2/2) \, d\tau$
  - Conserving generalized helicities: $K_{e,i} = \int P_{e,i} \cdot \nabla \times P_{e,i} \, d\tau$, $P = m u + q A$
  - Two distinct sets of surfaces in axisymmetric two-fluid equilibria:
    - **electron fluid:** $\psi = \text{const}$ (magnetic surfaces)
    - **ion fluid:** $\Psi = \psi + \varepsilon r u_\theta = \text{const}$; $\varepsilon = \ell_i/r_s$
  $\Rightarrow$ In a two-fluid Minimum Energy State (MES), ion and electron stream functions characterizing poloidal flows are **linear functions:** $\psi_e \sim \psi$, $\psi_i \sim \Psi$. This is analogous to constant $\lambda$ in a Taylor state.

* Steinhauer & Ishida, PRL 79, 3423 (1997)
Evidence of relaxation in an extremely high-\(\beta\) plasma state
- Flux conversion from toroidal to poloidal after super-Alfvénic translation
- Helicity preservation inferred by an interpretive model

Discovery of a new relaxed ST-like FRC state
- Large q and strong magnetic shear
- A broad core conforming to a Minimum Energy State

Confinement & stability of FRC-ST

Summary & conclusions
TCS facility at UW was built to investigate FRC formation and sustainment

- Translation of high-β energetic plasmoids formed by LSX/mod θ-Pinch.
- FRC formation and sustainment by Rotating Magnetic Fields (Hoffman, Milroy, this meeting).
High-β plasmoid was formed by LSX/mod θ-pinch, then ejected into confinement chamber at highly super-Alfvénic speed (~400 km/s).

FRC survived the extremely violent reflections, and retained a long lifetime.
High-β relaxation is evidenced by a poloidal flux amplification of the translated CT*

- Poloidal flux $\phi_p$ appears to increase significantly after 1st reflection, as inferred from external magnetic measurements:
  \[ \phi_p^{RR} = 0.31 x_s \Delta \Phi \]

- This has been verified, for the 1st time, by detailed internal probing.

- In an axisymmetric plasma, $\phi_p$ decays at a rate that depends on resistivity – it can never increase. The only way to amplify $\phi_p$ is by a topology-breaking, which is a marker of a turbulent relaxation.

Significant flux conversion occurs with magnetic helicity roughly conserved.

- Initial plasmoid has little $\phi_p$, strong $\phi_T$ at the ends.
- After violent reflections, it settles into a near-FRC state with $\phi_p$ increased significantly.
- Most of the oppositely directed $\phi_T$ annihilated, with only a reduced unidirectional $\phi_T$ remaining.
- Magnetic helicity was approximately conserved.

$\Rightarrow$ Significant toroidal rotation has been observed, but flow measurements are incomplete to determine whether the ion helicity is conserved.
Relaxation leads to a ST-like FRC state with a large q and significant shear

- High-$\beta \rightarrow B_{tor}/B_{pol}$ is small
- Large-$q$ arises from high elongation, $E$, as first noted by Ryutov.
- Strong shear is due to highly localized $B_\theta$.

\[ q \equiv \oint_{\psi} \frac{B_\theta d\ell}{2\pi r B_p} \sim \frac{2E}{\pi} \frac{B_{tor}}{B_{pol}} \]
FRC-ST is a new addition to relaxed states of magnetically confined plasmas.
A broad core of FRC-ST conforms to a two-fluid Minimum Energy State (MES)

In a two-fluid MES, ion and electron stream functions are linear functions, i.e., $\psi_i \sim \Psi$, $\psi_e \sim \psi$ (analogous to constant $\lambda$ in Taylor-states).

The inferred stream functions appear to be linear in a large core region, indicating that a broad core of FRC-ST is very close to a MES.

- MES core may promote good stability and confinement.

* Steinhauer, this meeting
FRC-ST exhibits up to 4x improvement in \( \tau_\phi \) with significantly reduced transport*.

Flux confinement time, \( \tau_\phi \)

- Well centered
- Conventional \( \theta \)-pinch FRC scaling

\[
\tau_{\phi}^{\text{LSX}} (\mu s) = 9 \times 10^3 \sqrt[3]{r_s \left\{ \sqrt{r_s (m)} / \sqrt{r_L (cm)} \right\}^{2.14}}
\]

Diffusivity, \( D_\perp = \eta_\perp / \mu_o \)

Dominant global instability observed in an FRC is $n=2$ rotating interchange mode

- $n=1$ tilt mode
  - MHD unstable, but
  - Not observed experimentally.

- $n=2$ rotating interchange
  - Driven by centrifugal force due to plasma rotation.
  - Usually stabilized by external static multipole fields in $\theta$-pinch formed FRCs.
  - Rotating Magnetic Fields also provide strong stabilizing effects.*

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FRC-ST exhibits remarkable stabilizing effects for n=2 rotating interchange mode

- θ-pincho FRC: n=2 is destructive
- Usually stabilized by external static multipole fields.
- FRC-ST: n=2 mode does not develop despite the absence of multipoles.
This is explained by a simple model taking into account magnetic shear.

- **Energy principle analysis (Bellan):**

\[
\delta W \approx \frac{1}{4} \int dV \xi_\perp^2 \left( B_{1\perp}^2 / \mu_0 + \Omega^2 r \nabla \rho \right)
\]

- neglecting \(J_\parallel\)-drive & curvature-related \(\nabla p\)-drive terms.

- **Stability criterion:**

\[
\left( \frac{d\rho}{d\Psi} \right)^2 > \frac{3R^2 Z_0 V_A^2}{n^2 V_A^2} \left| \frac{\nabla \rho}{\rho_0} \right| \sim 1, \text{ for } n = 2
\]

- **From inferred }q\text{ profile:**

\[
\left( \frac{d\rho}{d\Psi} \right)^2 \sim 5
\]

\(\Rightarrow\) **Sufficient to stabilize centrifugally driven }n=2\text{ mode.**
Summary & conclusions

Relaxation in an extremely high-β plasma has appeared in the TCS translation experiment, producing a new ST-like FRC state, *i.e.*, clearly not a Taylor state, strongly suggesting that a more general principle regulates the relaxation.

Modeling using the newly developed nearby-fluids theory shows that a broad core of FRC-ST is very close to a two-fluid Minimum Energy State (MES).

This relaxed plasma state exhibits significantly reduced transport with up to 4x improvement in confinement.

FRC-ST also exhibits a remarkable stabilizing property for the normally lifetime terminating n=2 centrifugally driven interchange modes in θ-pincho FRCs, which is attributed to the strong magnetic shear.