Feedback stabilization of tearing modes in RFPs with a resistive wall above the ideal wall limit

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Known: resistive wall modes can be unstable when perfectly conducting (PC) wall stabilizes

Earlier work: feedback: sensing $\tilde{B}_r$ or $\tilde{B}_\theta$ (normal or tangential)

Tokamaks or RFPs :: increasing or decreasing $q(r)$ profile

New: we sense a combination of $\tilde{B}_r$ and $\tilde{B}_\theta$

New: stability of tearing modes is possible ABOVE the wall stabilization regime. For ideal modes? No.

Related to virtual wall of Bishop (1989)? For $Re(\omega) = 0$, it is qualitatively related. Otherwise no.
• Applications: stabilizing the tearing modes in RFPs for single (or quasi-single) helicity operation in RFPs; controlling amplitude of $m = 0$ modes to weaker coupling of $m = 1$ modes; possibly control the amplitude of neo-clasical tearing modes in tokamaks.
RADIAL AND TANGENTIAL SENSING (REVIEW)


✔ It appears to work better because it is less sensitive to $m \rightarrow m \pm 1$ coupling.

✔ Works better in DIII-D too.
MODEL

Cylindrical tokamak model, reduced \( \left( \frac{R}{a} \gg 1 \right) \) resistive MHD equations, resistive wall (RW) at plasma edge, control flux (single \( m, n \)) applied at \( r_c \)
Resistive wall tearing mode – reduced MHD

\[ \mathbf{B} = \nabla \psi \times \hat{z} + B_0 \hat{z} \quad \mathbf{j} = j_z \hat{z} = -\nabla^2 \psi \]

\[ \mathbf{B} \cdot \nabla j_z + \mathbf{\tilde{B}} \cdot \nabla j_z = \rho \omega / dt \quad \nabla^2 \tilde{\psi} - \frac{m}{r} \frac{j'_z}{k \cdot \mathbf{B}} \tilde{\psi} = 0 \quad \text{vac.} \]

Matching at \( r = r_s \) (\( q = m/n \)) – visco-resistive ct. psi tearing matching condition:

\[ [\psi']_{rt} = \gamma \tau_t \psi(r_t) \quad \tau_t \sim \mu^{1/6} / \eta_p^{5/6} (k \cdot \mathbf{B})^{1/3} \]

Matching at \( r = r_w \) – resistive wall – thin wall (ct. psi) matching condition:

\[ [\psi']_{rw} = \gamma \tau_w \psi(r_w) \quad \tau_w \sim r_w \delta / \eta_w \]
FEEDBACK MODEL

Feedback – linear combination of radial field \( \psi(r_c) = -G\psi(r_w) \) and poloidal field \( \psi(r_c) = K\psi'(r_w-) \) \([\tilde{B}_r = i m\psi/r; \tilde{B}_\theta = -\partial\psi/\partial r]\)

\[
\psi(r_c) = -G\psi(r_w) + K\psi'(r_w-)
\]
$\Delta_1 < 0 \implies$ TM is stable with PC (perfectly conducting) wall at $r_w$
\[ \Delta_1 > 0 \quad \implies \text{TM is unstable with PC wall at } r_w \]

\[
\begin{bmatrix}
\Delta_1 - \gamma \tau_t & l_{12} & 0 \\
\tau_t & \Delta_2 - \gamma \tau_w & l_{23} \\
-Kl_{21} & -G + Kl_{22}^{(-)} & 1
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{bmatrix} = 0
\]

\[
\begin{bmatrix}
\frac{\Delta_1}{\tau_t} - \gamma \\
\frac{l_{12}}{\tau_t} \\
\frac{l_{21}(1-Kl_{23})}{\tau_w} \\
\frac{\Delta_2 - l_{23}(G-Kl_{22}^{(-)})}{\tau_w} - \gamma
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2
\end{bmatrix} = 0
\]
Stability conditions

G = 1.5

$\Delta = 0.5$
Stability regimes in $G, K$

$\tau_w/\tau_t = 1$: Real marginal ... C.c. marginal ... Stable region

Always a stability window! Thinner, with large $G, K$, for large $\Delta_1$. For $\Delta_1$ large enough, $G$ and $K$ are both required.
The effect of $\frac{\tau_w}{\tau_t}$
**The effect of** $\tau_w / \tau_t$:

Making resistive wall thicker or less resistive ($\tau_w$ larger)

- $\tau_w / \tau_t$ helps in the wall stabilization regime ($\Delta_1 < 0$). Slow penetration of flux from mode in the plasma slows mode.

- Harms above wall stabilization regime ($\Delta_1 > 0$). Prevents penetration of the feedback flux.
Feedback based on tangential field outside the resistive wall

\[ \psi(r_c) = -G\psi(r_w) + K\psi'(r_w+) \]

No stabilization above \( \Delta_1 = 0 \), where tearing mode is wall stabilized.
Ideal plasma nonresonant external modes – mode rational surface in vacuum. Non-resonant RFP modes

Wall stabilization: $1.5 < \hat{\beta} < 2.0$
Unstable $\hat{\beta} > 2$
RFP-like $q$ profiles – tearing

delta1=−0.5 RFP

delta1=−0.25 RFP

delta1=−0.01 RFP

delta1=.25 RFP

delta1=.5 RFP

delta1=1.0 RFP
Finally, ideal plasma resonant modes (RFP)

Plasma resistive layer at $r = r_t$

Jump in $j$ or in $p$ at $r = a$

Resistive wall at $r = r_w$

Control at $r = r_c$

\[
\begin{bmatrix}
    \Delta_1 - \gamma\tau_t & l_{12} & 0 \\
    l_{21} & \Delta_2 + \frac{\beta}{1 + \rho\gamma^2/F_a^2} & l_{23} \\
    0 & l_{32}(1 - l_{34}K) & \Delta_3 - \gamma\tau_w - l_{34}(G - l_{33}^{(-)}K)
\end{bmatrix}
\begin{bmatrix}
    \alpha_1 \\
    \alpha_2 \\
    \alpha_3
\end{bmatrix} = 0
\]
No possibility of stabilization for \( \hat{\beta} > 2 \) (above wall stabilization)
Summary

- Tearing modes (tokamak or RFP): linear feedback sensing radial magnetic field and tangential field can stabilize the tearing mode below and above the threshold for stability with a PC wall. Some relation with virtual wall of Bishop ('89), when $Re(\omega) = 0$.

- Increasing $\tau_w/\tau_t$ (making wall more conducting) increases the range $(K, G)$ of stability for $\Delta_1 < 0$ ... decreases(!) the range for $\Delta_1 > 0$. Easy to understand.

- Using radial field and external tangential field can stabilize for $\Delta_1 < 0$ but not for $\Delta_1 > 0$.

- Resonant or non-resonant ideal modes: can stabilize below threshold for stability with PC wall, but not above.

- Applications: stabilizing all the $m = 1$ modes in RFPs but one – single helicity or quasi-single helicity; controlling the amplitude of $m = 0$ modes to
weaken the coupling of $m = 1$ modes; perhaps controlling the amplitude of neoclassical tearing modes in tokamaks.