Hartmann Flow Physics at Plasma-Insulator Boundary in the Maryland Centrifugal Experiment (MCX)

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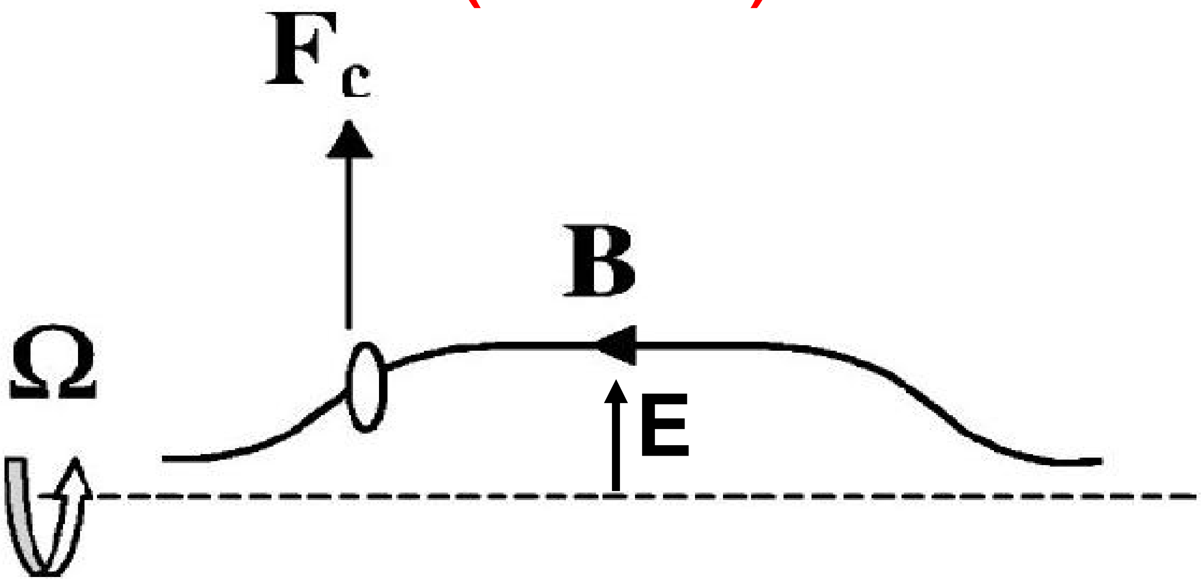
Maryland Centrifugal Experiment (MCX)



- A rotating mirror system.
- Shear rotational flow stabilizes instabilities and provides centrifugal force confining the plasma towards the mid-plane.

Frictions on Rotational Flow?

For the MCX idea to be practical for generating energy, the energy required to maintain the supersonic rotational flow should be “small.”

Considerations:

- Neutrals Drag?
- Hartmann Friction?

Parameter of concern:

Momentum confinement time, τ_{mom}

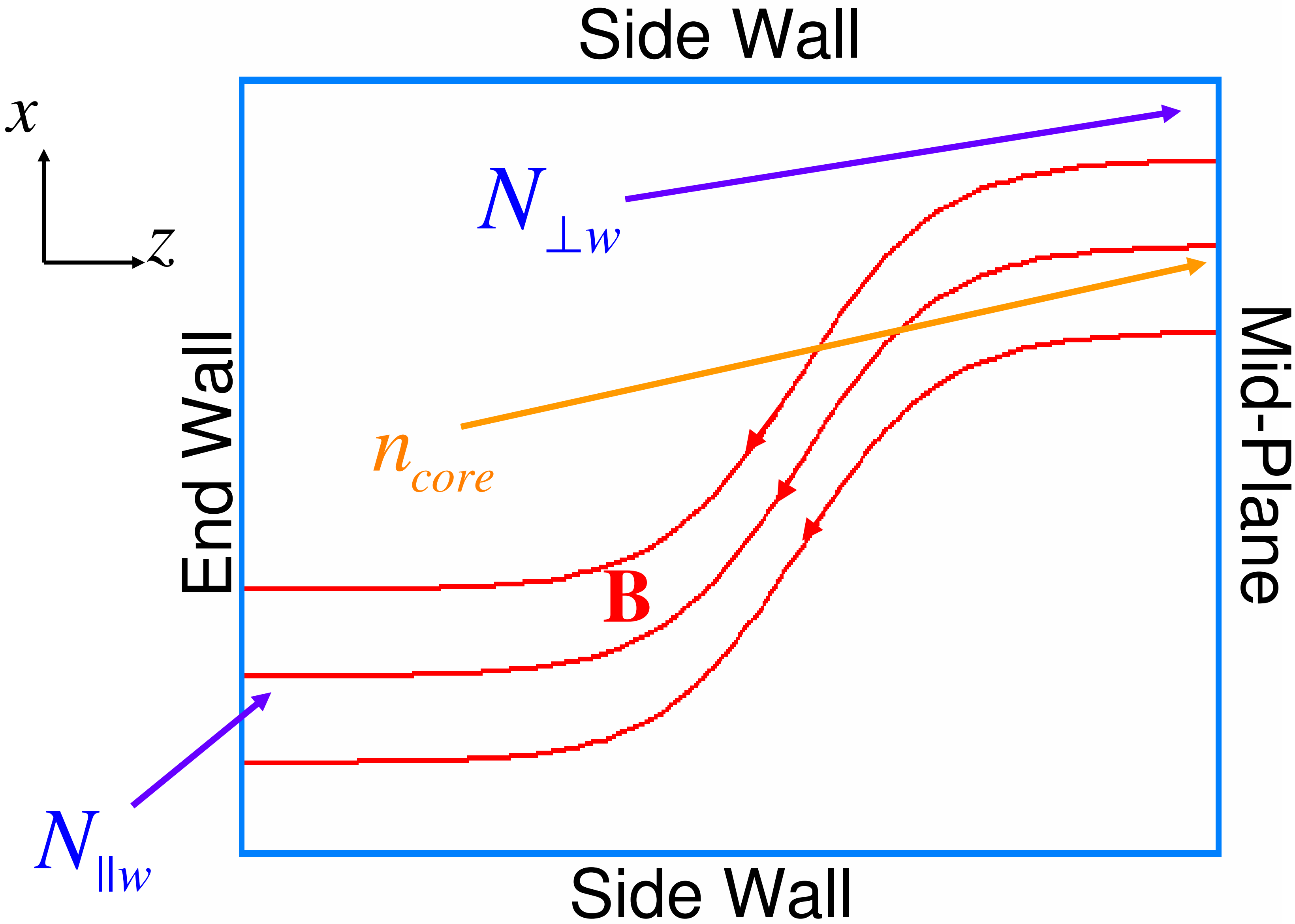
The Role of Neutrals

- Limited existing works for centrifugal confined plasma: Open field lines and existence of centrifugal force.

- We would like to know what role neutrals are playing in the momentum confinement time, τ_{mom}.

- We will begin our study by considering the neutrals distributions in the system with ionization, charge exchange with prefect recycling.

The Geometry



Normalized Model Equations

$$(nu)' = Nn$$

$$(nu^2)' = -\gamma n' - \gamma nN(u-U) + NnU \boxed{+ng}$$

$$(NU)' = -Nn \qquad (1)$$

$$(NU^2)' = -\gamma N' + \gamma nN(u-U) - NnU$$

$$g \equiv (u_\perp^2/2)', \ \gamma = \alpha_{CX}/\alpha_I, \ T_i = 0 \text{ and } T_e = T_N$$

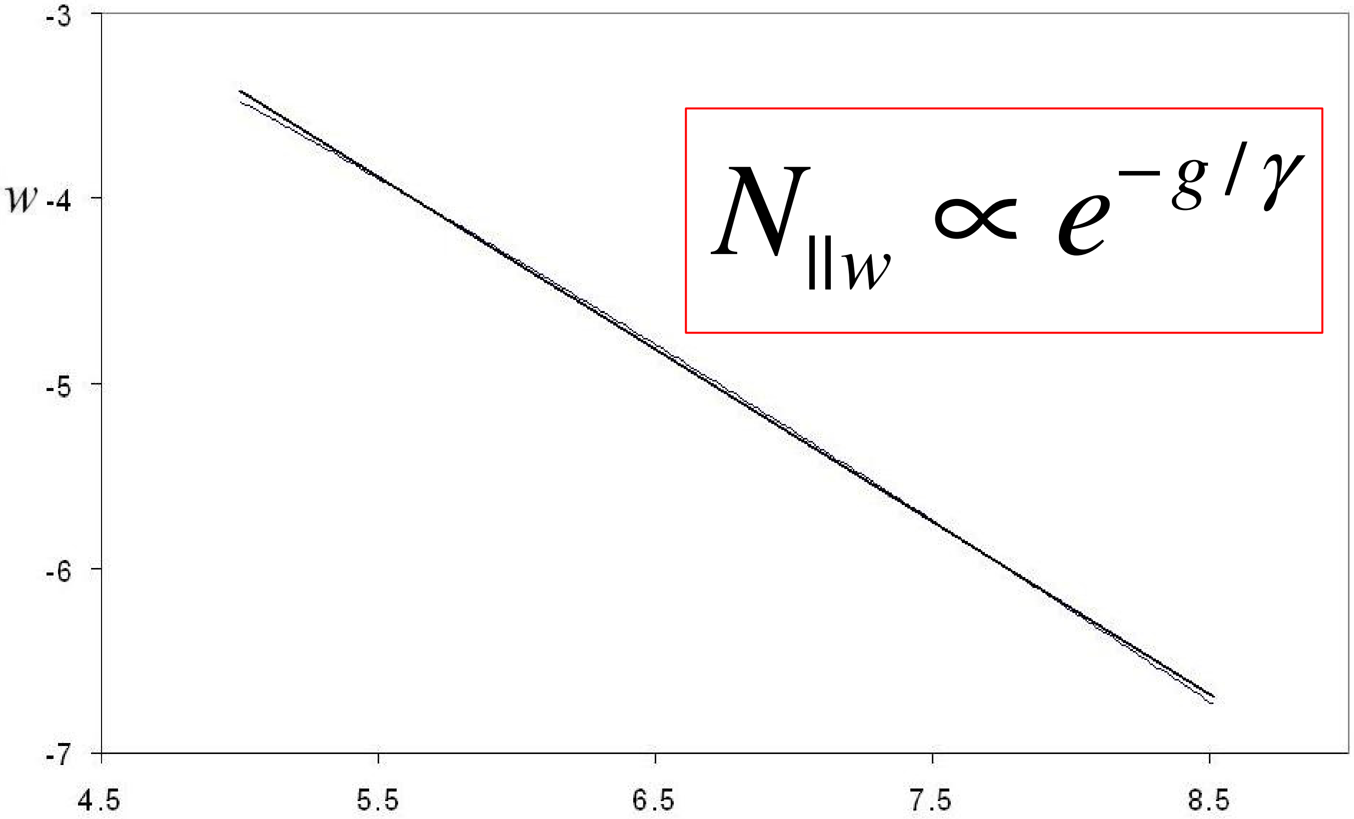
Normalization:

$n_o = n_{ave}$, $t_o = 1/\alpha n_{ave}$, where $n_{ave} = \int (n+N)dV/V$

$l_{cx,i} = c_s/(\alpha_I \alpha_{CX} n_\infty^2)^{1/2} \sim 3\text{cm to } 5\text{cm}$ ← Geometrical mean of λ_{CX} and λ_{ion}

Wall Neutral Density vs. Confinement Force

1D numerical result



$\ln N_{\|w}$ vs. g/γ

$$N_{\|w} \propto e^{-g/\gamma}$$

In MCX, $g/\gamma \sim \left(M_s^2/2\right)$

Rotation Mach number

Cross-field 1D Solution

- It can also be shown that[1],

Neutral density at Wall

Plasma classical cross-field diffusion coefficient, $\eta n T/B^2$

Plasma core density

$$\frac{N_{\perp w}}{n_{core}} \approx \frac{D_{\perp}}{2D_N} \ll 1$$

Neutral penetration depth is also $\sim l_{cx,i}$

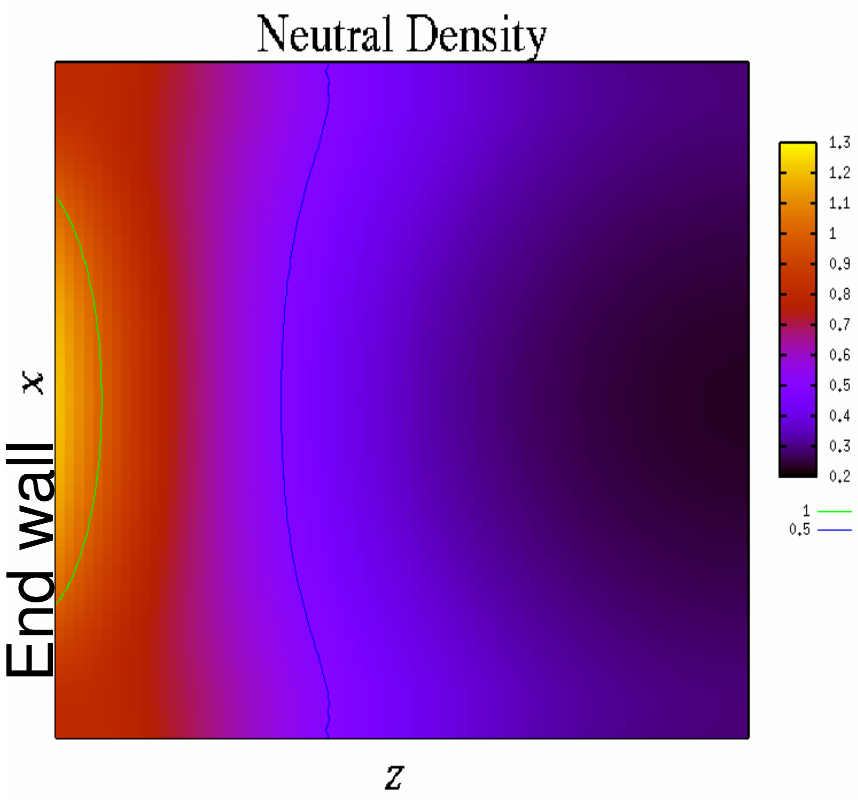
Neutrals diffusion coefficient, $v_{t,i}^2/\alpha_{CX} n$

[1] R J Goldston and P H Rutherford 1995, *Introduction to Plasma Physics,* Institute of Physics Publishing, Philadelphia.

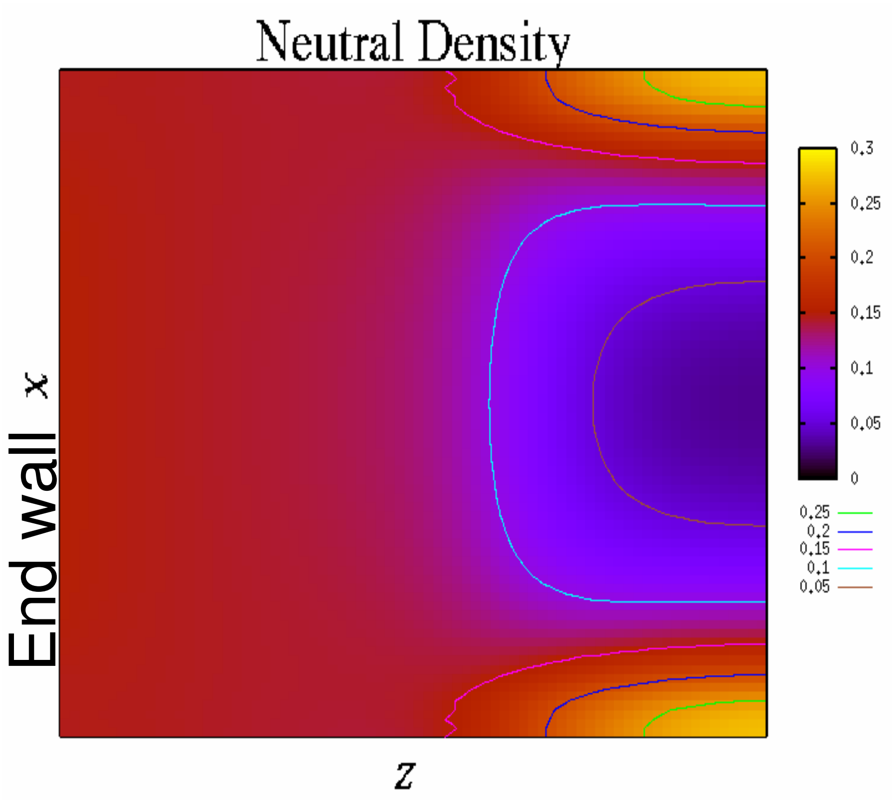
Numerical Calculations in 2D system$

$L_x = L_z = 1$, artifical confinement
force acting towards right in z.

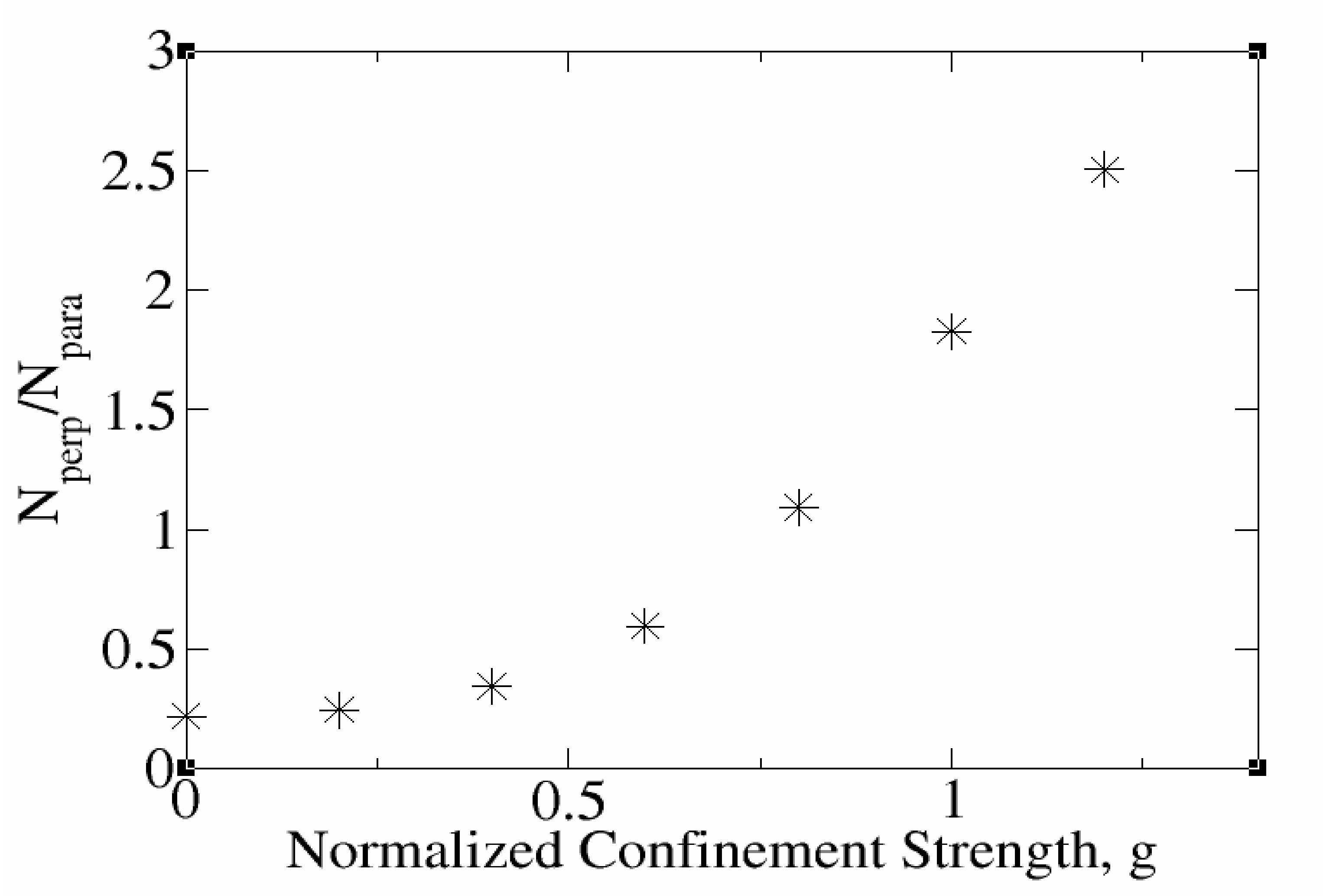
g=0, no confinement



g=1, with confinement



2D numerical results: Ratio of Neutrals densities at different walls



Remarks on Neutrals Distributions

- $N_{\perp w} << 1$ and $N_{\parallel w} \sim O(1)$ in 1D separately.

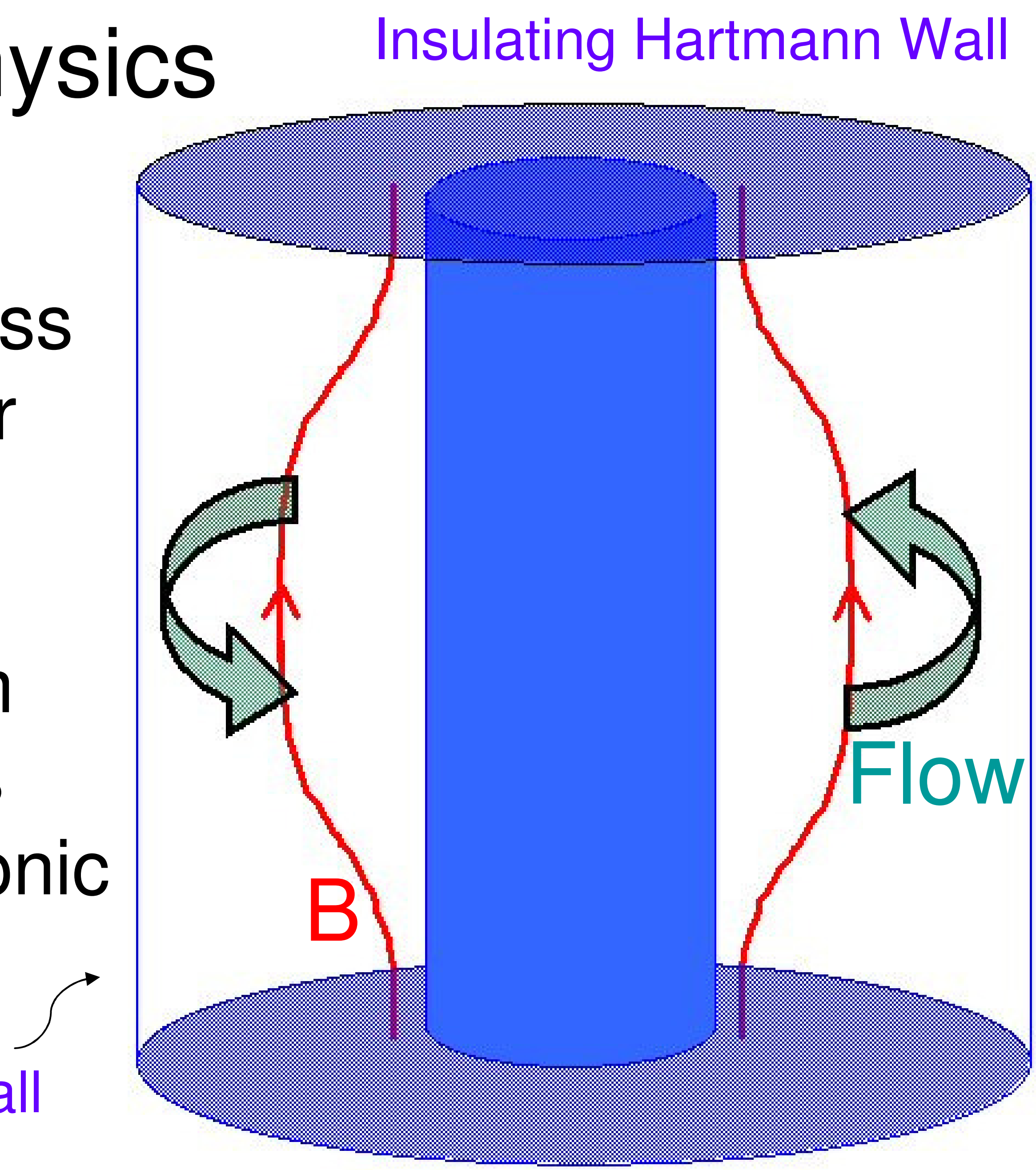
- $N_{\parallel w} \propto \exp(-M_s^2 / 2)$ in 1D if confinement (i.e. rotation) is considered

- $N_{\perp w} / N_{\parallel w}$ increases as better confinement is achieved in 2D.

Hartmann Physics

- Main Flow Across the Strong Mirror Field

- Top and Bottom Hartmann layers Prevent Supersonic Flow?



Classical 1D Hartmann Results[%]

Given B_o, driving force density F ,system size L_z ,viscosity μ and resistivity η.

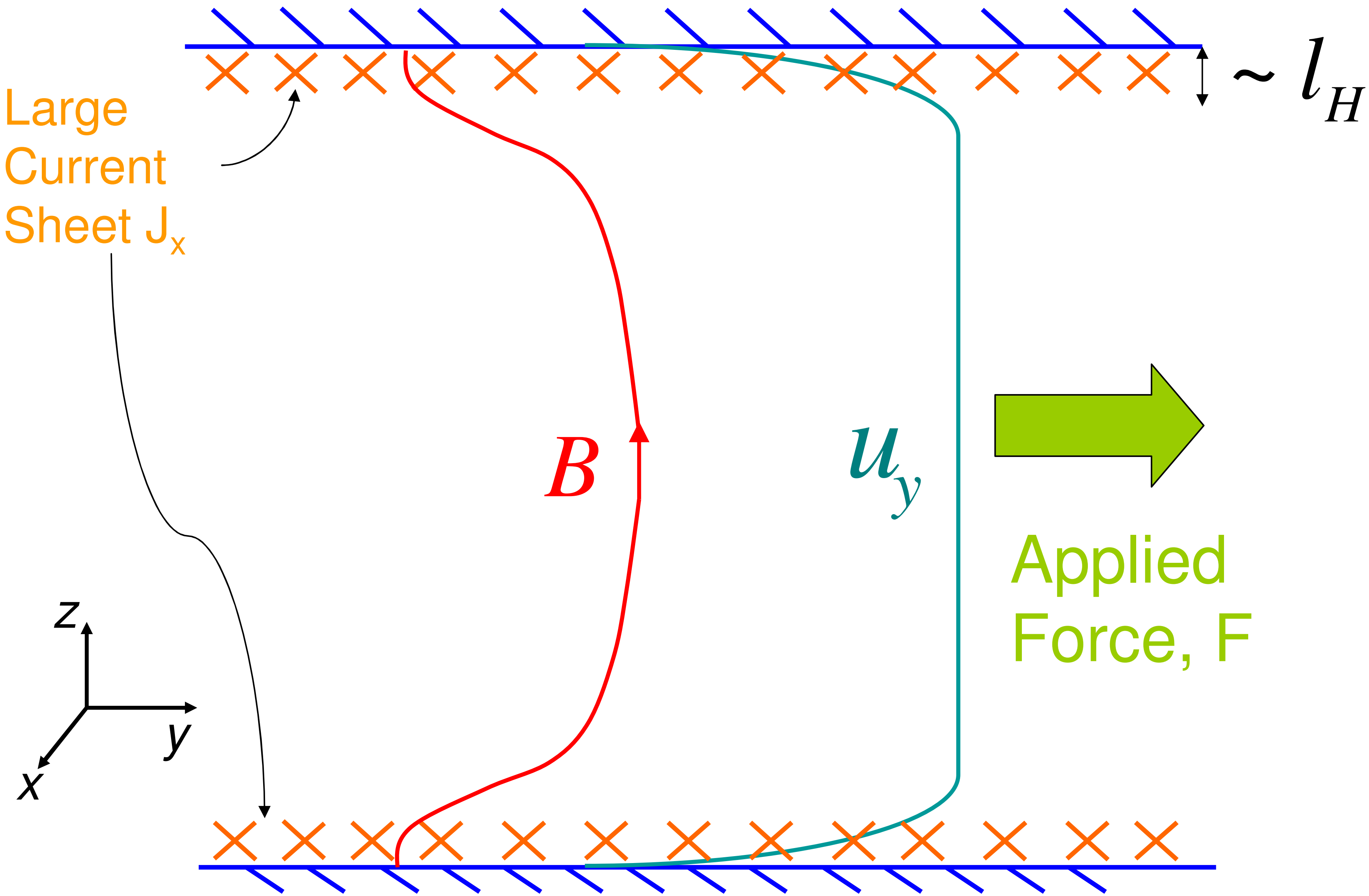
Hartmann Number: $H_a \sim B_0 L_z / \sqrt{\mu\eta}$

Hartmann Layer width: $l_H \sim L_z / H_a$

Maximum flow : $u_{y,core} \sim \dfrac{FL_z^2}{2\mu H_a}$

Momentum confinement time: $\tau_H \equiv \dfrac{nMu_{y,core}}{F/2} \sim \dfrac{\tau_\mu}{H_a}$

[%]J. D. Jackson, "Classical Electrodynamics", 2nd ed., (Wiley, New York, 1975).



Large Current Sheet J_x
$\sim l_H$
B
u_y
Applied Force, F
z
y
x

Hartmann numbers in MCX

If we substitute the MCX parameters* into the formula, we have $H_a \sim 10^6$, $l_H \sim 10^{-4}$ cm and $\tau_H \sim 10^{-5}$sec. Therefore by considering the conventional Hartmann physics alone, the idea of MCX is not feasible for generating energy because too much energy is required to support a supersonic flow (smalless of τ_H).

Neutrals Effects on Hartmann

- One of the main physical parameters in conventional Hartmann problem is the resistivity η .
- The effect of neutrals on η can be estimated from the ratio of e-n and e-i collision frequencies:

$$\nu_{en} / \nu_{ei} \sim 3\times10^{-3} \left(N / n \right)$$

which might not be significant because the confinement limits the neutrals densities at the wall.

Boundary Plasma Density Effect

In the classical Hartmann calculation, the plasma density is a constant. Yet, a simple analysis shows that, by taking $\mu = n^m \tilde{\mu}_m$ (i.e. μ is Braginskii's η_1 for $m = 2$), we have

$$u_{y,core} \sim \frac{1}{n_{wall}^{m/2}} \sqrt{\frac{\eta}{\tilde{\mu}_m}} \frac{FL_z}{B_o}$$

Therefore, mechanism that lowers the boundary plasma density could increase the core flow and thus the momentum confinement time, τ_{mom}.

Such mechanisms could be (i) plasma outflow (recycling) or (ii) confinement.

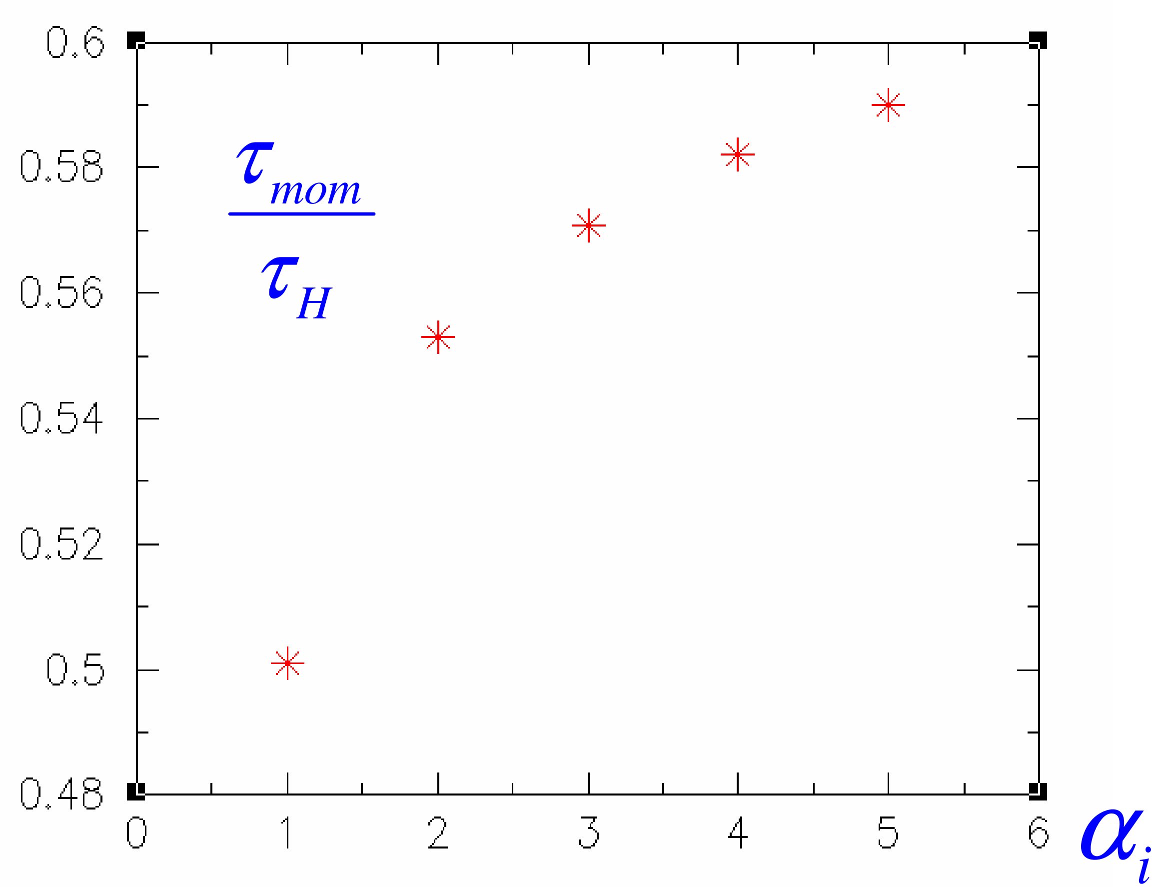
- (i) For the recycling effects, we can exam. the change in $\tau_{\rm mon}/\tau_{\rm H}$ as α_i changes.

Redefine

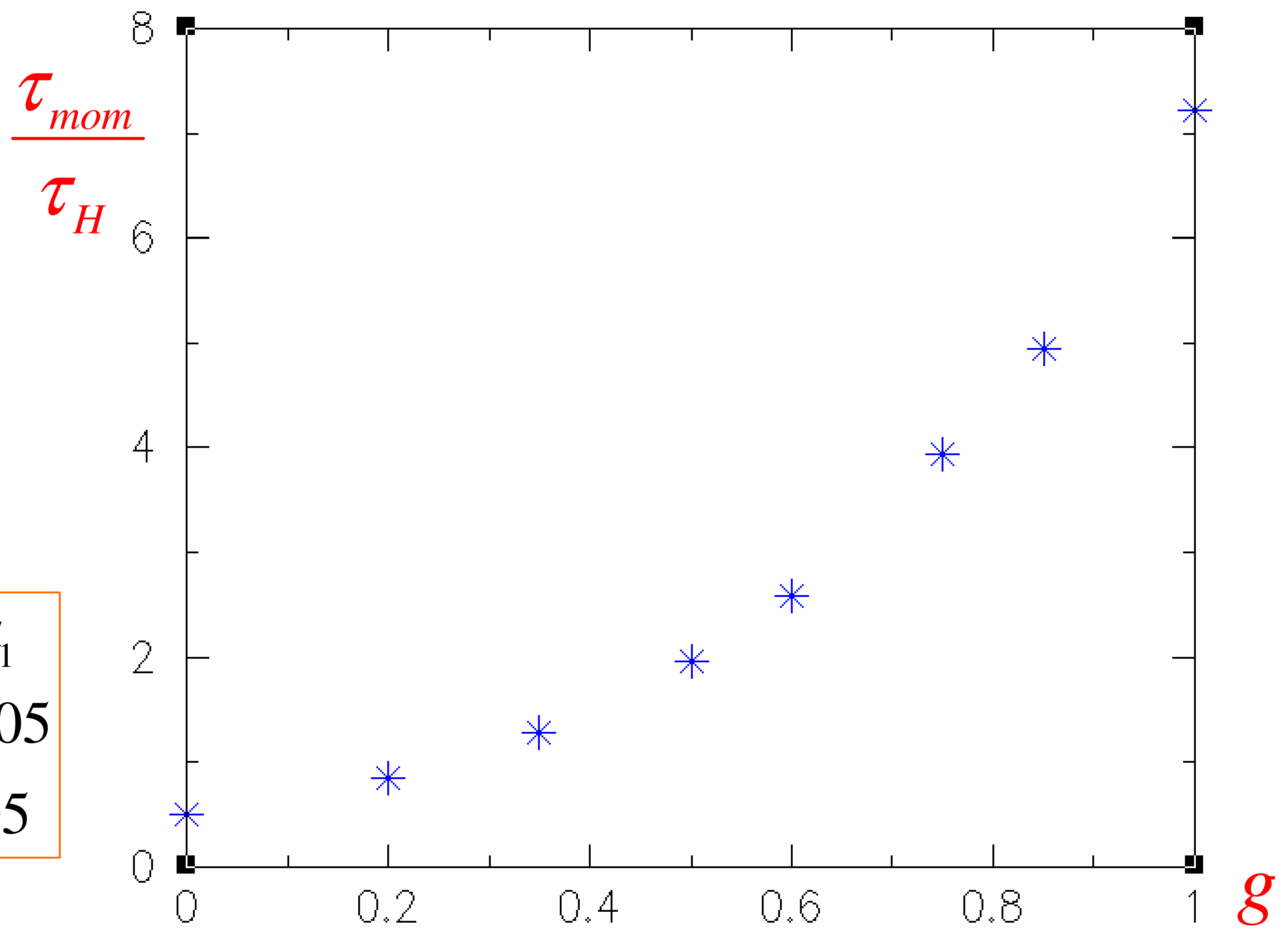
$$\tau_{mom} \equiv \frac{2\int p_y \mathrm{dz}}{F\int \mathrm{dz}}$$

$$\mu = n\tilde{\mu}_1$$
$$\tilde{\mu}_1 = 0.05$$
$$\eta = 0.05$$

*Note that part of the total momentum goes to the neutrals.



- (ii) Putting an artificial confinement force along z towards the mid-plane, the τ_{mom} is found to be increased with g as expected.



$\mu = n\tilde{\mu}_1$

$\tilde{\mu}_1 = 0.05$

$\eta = 0.05$

Selected Profiles along z with different confinement strengths ($n_{ave} = \int (n+N)dV/V = 1$)

— g=1

— g=0.5

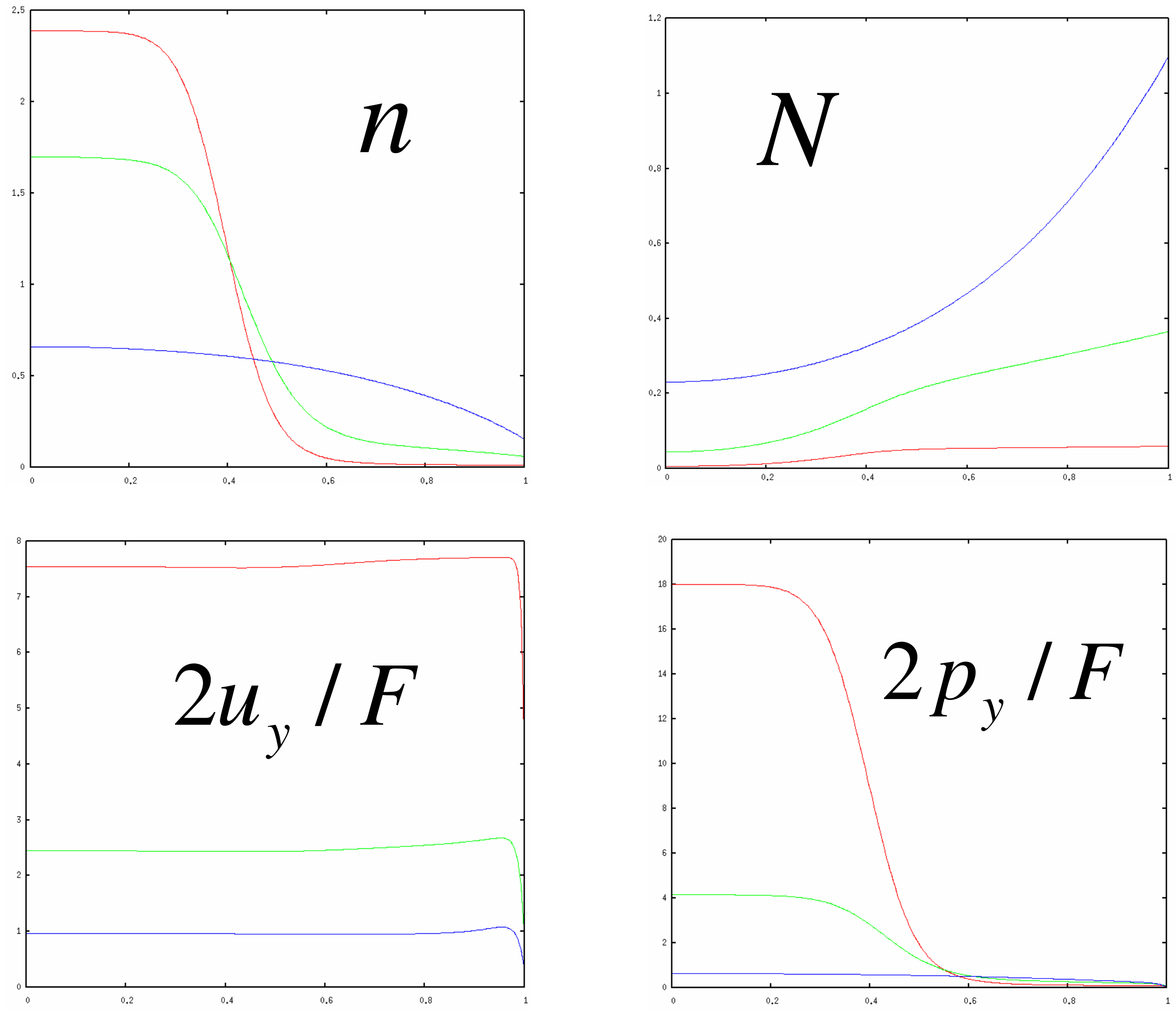
— g=0

$\mu = n\tilde{\mu}_1$

$\tilde{\mu}_1 = 0.05$

$\eta = 0.05$

Force acting to the left (mid-plane)



Hall Effects

The existence of the thin current sheet inside the Hartmann layers leads to the consideration of adding Hall terms to the Ohm's Law:

$$E = u \times B + \eta j + \frac{j \times B - \nabla p_e}{ne}$$

It can be shown analytically that the Hartmann layers are broadened and the flow is increased with a fixed driving force. The contribution of the hall effect can be measured from the parameter $\varepsilon \equiv c / \omega_{p,i} L_z$.

Since for MCX[*],

$$10^{-2} \sim \varepsilon >> \eta \sim 10^{-5}$$

after some normalizations, the hall effect could be significant in the Hartmann layers and it can be shown that

$$\tau_{H,hall} \sim \frac{\tau_\mu}{\left|H_{a,hall}\right|} >> \frac{\tau_\mu}{H_a} = \tau_H$$

However, secondary flow in the *x*-direction is being generated too. That might be a concern for MCX.

Profiles with Hall effects

It can be shown that

$$\tau_{hall} \propto \sqrt{\varepsilon / \mu}$$

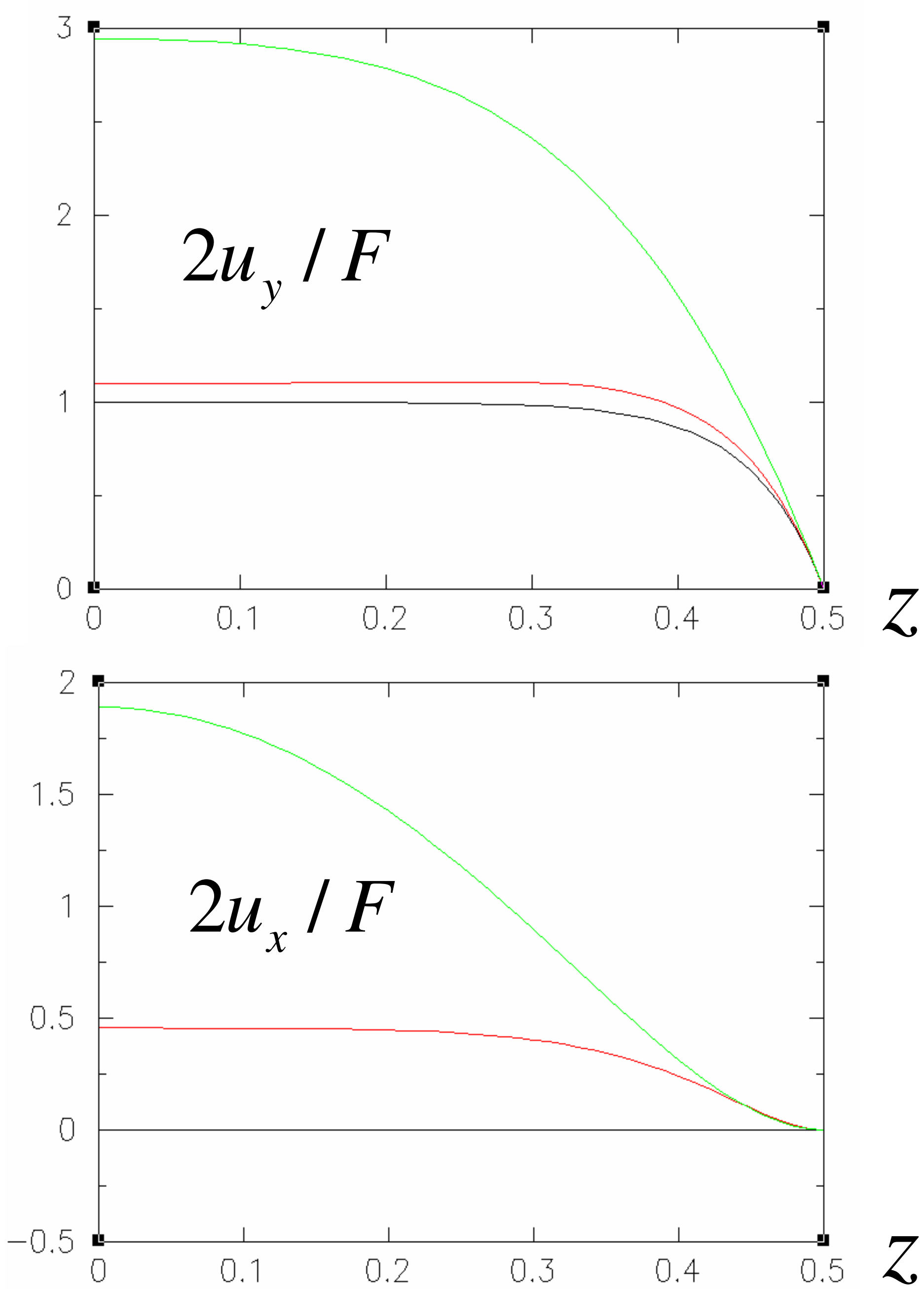
when $\varepsilon >> \eta$.

$$\eta = \mu = 0.05$$

— $\varepsilon = 0$

— $\varepsilon = 0.05$

— $\varepsilon = 0.5$



Conclusions

- Supersonic azimuthal flow is required in MCX.
- Neutrals density is small at the Hartmann wall when confinement is good (and recycling is the only source). Thus drag from neutrals should be limited.
- Classical Hartmann layer limits core flow speed.
- Small plasma density at the wall loose this restriction. Good confinement helps.
- Hall effects around Hartmann layer help further. But generate secondary flows.

Future Works

- Analysis in full geometry is required especially for the Hall effects secondary flows.

- Since both plasma and neutrals densities are small around the walls, kinetic effects (e.g. in η and μ) might need for further analysis.

Appendix

*MCX Parameters

$$B_o = 1 \text{ T}, T = 10 \text{ eV}, n_{core} = 10^{20} \text{ m}^{-3}, L_z = 1 \text{ m}, a = 0.3 \text{ m}$$

$Numerical simulations parameters

The 2D simulation parameters are normalized based on the MCX parameters above.

	2D simulations	***Real System (MCX)***
α_I	1	~ 0.5
α_{CX}	9	~ 1
$l_{cx,i}$	≈ 0.3	~ 0.04
$\beta' \equiv p / B^2$	$0.025 - 0.25$	~ 0.15
η, μ	$0.01, 0.01$	$\sim 2 \times 10^{-5}, 3 \times 10^{-8}$
$g \approx u_\perp^2 / 2$	$0 - 1.2$	$2 - 8$