Numerical Studies of Line-Tied Kink Modes
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Objective

This study seeks a quantitative description of the nonlinear saturation of line-tied kink modes, including effects on the mean-field current profile and dynamic properties.

Outline

1. Introduction--*motivations, configurations studied, NIMROD modeling*
2. Relaxation study--*relaxation principle, generic modeling*
3. Rotating Wall Machine modeling--*description, simulation*
4. Helical current injection--*Pegasus application, modeling*
5. Summary
Introduction

• Kink-mode relaxation of current profiles occurs in many plasma configurations with line-tied magnetic field: ‘stabilized’ pinches, spheromaks, DC injection for current drive, astrophysical jets (?), …

• Theory:
  • Relaxation (Taylor ‘74) is a conceptual start.
  • Driven-damped systems require further analysis.
    • Jensen and Chu (‘83 and ‘84) considered relaxed state with injection.
    • Turner (‘84) found relaxed flux-core-spheromak equilibria.
    • Taylor (‘86) suggests a transition to nonsymmetric $B$ with uniform $\lambda (= \mu_0 J \cdot B / B^2)$ when driven through the first eigenvalue.
    • Relaxation through ‘plasma turbulence’ is suspect, however, when the most unstable mode of the symmetric pinch is global, regardless of $S (= \tau_I / \tau_A)$.
  • Montgomery and coworkers considered the applicability of minimum dissipation (‘88 and ‘90).
Introduction (continued)

• Our earlier spheromak simulations (with Finn and del Castillo Negrete) with simple 0-\(\beta\) resistive MHD reproduced several important features of driven spheromaks:
  • relaxation of mean \(\lambda\) profiles, amplification of bias flux, dynamo sustainment, different classes of saturated states, chaotic scattering of \(B\), and flux-surface formation upon decay.
  • Nonlinear saturation of the \(n=1\) kink-tearing is central to all of these features.
• SSPX evolution has been modeled (with Cohen and Hooper) with temperature evolution and anisotropic \(T\)-dependent transport coefficients.
  • Important MHD/transport interaction during transients leads to good confinement.
  • The familiar \(n=1\) kink activity is responsible for formation.
  • Voltage spikes during formation result from \(n=1\)-excited reconnection and subsequent inductance jumps (Hooper+ ‘05)
Introduction (continued)

- Other pinch experiments have observed kink mode saturation.
  - Current-trajectory scattering--Colgate & coworkers (early ‘60s) for example
  - Rotating Wall Machine (RWM) by Forest and coworkers also produces a kink instability--related to ideal external kink threshold (Bergerson+ ‘05).

- Noninductive startup of spherical tori has practical value.
  - Symmetric injection studied on HIT, NSTX. (Jarboe, Nelson, Redd, Raman+)
  - Localized injection studied on CDX, CCT (Ono, R. Taylor+)
  - Injection via miniature plasma guns on Pegasus is recent (see Eidietis poster)
The NIMROD code is used for our numerical computations of basic resistive MHD and MHD+collisional transport.

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \mathbf{J}) \quad \text{Faraday’s/Ohm’s laws}
\]

\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B} \quad \text{low-}\omega \text{ Ampere’s law}
\]

\[
\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \nu \rho \nabla \mathbf{V} \quad \text{flow evolution}
\]

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = \nabla \cdot D \nabla n \quad \text{particle continuity}
\]

\[
\frac{n}{\gamma - 1} \left( \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -\frac{p}{2} \nabla \cdot \mathbf{V} + \nabla \cdot n \left[ \chi_\parallel \hat{b} \hat{b} + \chi_\perp (\mathbf{I} - \hat{b} \hat{b}) \right] \cdot \nabla T + \frac{\eta J^2}{2} \quad \text{(single) temperature evolution}
\]

\[
\hat{b} \equiv \mathbf{B}/|\mathbf{B}| \quad \text{local magnetic direction vector}
\]

- Braginskii transport coefficients are used for \(\chi_\parallel\) (electron), \(\chi_\perp\) (ion), and \(\eta\).
- The NIMROD code [http://nimrodteam.org] evolves the system in 3D.
  - High-order finite elements help resolve anisotropies [JCP 195, 355 (2004)].
Relaxation Study

MINIMUM DISSIPATION YIELDS TWO $\lambda$’s


1. For steady state, minimize $\int dx \, \eta j^2$ in $dE_{\text{MAG}}/dt = IV - \int dx \, \eta j^2$ at constant $\int dx \, E \cdot B$ in $dK/dt = 2(\nabla \Phi - \int dx E \cdot B)$ (once with $E = - \nabla \phi$, once with $E = \eta j$):

$$\eta \nabla \times j = \lambda (\nabla \phi + \eta j)$$

a. Inside the separatrix, drop $\nabla \phi$:

$$\eta \nabla \times j = \lambda \eta j \rightarrow \nabla \times B \approx \lambda_o B$$

$$K \approx 2V \Phi \tau \quad \tau = \left(\mu_o/2\lambda_o^2 \eta\right)$$

b. Inside the flux core (open lines, same $\eta$):

$$\nabla \phi + \eta j \approx 0 \quad \lambda = \lambda_{\text{GUN}} = \mu_o I/\Phi$$

$$IV \approx \int_{FC} dx \, \eta j^2 = (\eta L/\pi a^2) I^2$$

2. Can be stable in steady state despite large $\nabla \lambda$ at separatrix. Power distributes to offset resistive loss for all modes.
Minimum Dissipation (continued)

TWO $\lambda$‘s YIELD TWO CIRCUITS

1. Model describes a system of open field lines in 3D whose 2D projection consists of:

   (1) A flux core connected to the gun, with $\lambda = \lambda_{\text{GUN}} = \mu_0 I / \Phi$ (current I, bias flux $\Phi$)

   (2) An initially empty separatrix region, enveloped by the flux core and its return flux, in which magnetic relaxation creates a closed spheromak (projection) with $\lambda = \lambda_o \approx 2/a$ when fully relaxed, with a the minor radius (see sketch at right).

2. In the flux core: $IV = \int \eta \cdot j = E_{\text{MAG}} / \tau$

   Inside separatrix: $2V \Phi = 2\int \eta \cdot B = (\psi_{\text{POL}} \psi_{\text{TOR}}) / \tau$

   Equate $V = I^{-1} \int \eta^2 (\text{flux core}) = \Phi^{-1} \int \eta \cdot B$ (sepa.)

   The $\eta$‘s are the same: all open field lines
Minimum Dissipation-Predictions

<table>
<thead>
<tr>
<th>Spheromak*</th>
<th>Tokamak Injection</th>
</tr>
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<tbody>
<tr>
<td>$\frac{\psi_{POL}}{\Phi}$</td>
<td>$(2V\tau/\Phi)^{1/2}$</td>
</tr>
<tr>
<td>$\frac{I_{TOR}}{I}$</td>
<td>$1 \rightarrow (\frac{\psi_{POL}}{\Phi})^{1/3}$</td>
</tr>
<tr>
<td>Threshold</td>
<td>$\lambda_{GUN}a \approx 2$</td>
</tr>
<tr>
<td>Bubbleburst</td>
<td>$\lambda_{GUN}r_{GUN} \approx 2$</td>
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*Fits NIMROD 2000, applied-voltage simulation results.
Recent spheromak computations for the minimum-dissipation model specify injected parallel current.

- Simple 0-$\beta$ resistive MHD at $S$ of $O(10^3)$ but $Pm=1$.
- Cylindrical geom. with bias field penetrating electrodes at top and bottom.

Uniform bias flux with specified net $<\lambda_{\text{inj}}>=7.3$ ($\mu_1=4.37$ at $H/R=1.5$) produces ~150% flux amplification.

Saturation at $S=5K$, $Pm=1$ is steady, but the boundary layer allows a current-short.
Specifying the $<\lambda_{inj}>$ profile avoids short-circuiting.

- The nonuniform $<\lambda_{inj}>$ profile matches the 1D paramagnetic pinch for Ohmic/force-balance consistency.
- A thin resistive layer along the electrodes (without extra viscosity) allows a clean injection.
- *(Somewhat preliminary)* computations with central $<\lambda_{inj}>$ of 4.5-5.5 show a sharp transition to limit cycles at very low flux amplification (~30%). [S~1500, Pm=1]

**Evolution of mag. fluctuation energies with central $<\lambda_{inj}>$=5.125.**

**Evolution of mag. fluctuation energies with central $<\lambda_{inj}>$=5.1875.**
Steady saturated state clearly shows $n=2$ structure.

- The dominant $n=1$ first kinks the current column.
- Significant $n=2$ structure appears at saturation just below transition.
- With the fixed injection profile, the peak $\lambda$ exceeds the central $<\lambda_{\text{inj}}>=5.125$. 
Above the transition point in $<\lambda_{\text{inj}}>$, there are indications of a secondary instability that destroys the double corkscrew.
Spreading of current is evident from constant-\(\phi\) slices.

- Sequence of constant-\(\lambda\) contours at \(\phi=\pi\) (in the -x direction).
Toroidal averaging removes much of the fine structure.

- Sequence of constant-$\langle \lambda \rangle$ contours (toroidally averaged).

- The between-crash phase ($t=0.3431$) bears resemblance to the $2-\lambda$ profile of the minimum dissipation theory.
Kink Mode in the RWM

• The first series of experiments do not have the rotating wall, and the evolution of the kink is studied.
• An array of plasma guns launch current along an axial bias field. Bergerson et al. (PRL ‘05) relate observations of fluctuations to ideal external kink stability.
• Recent NIMROD MHD simulations of RWM use realistic parameters:
  – $n_{\text{column}} \sim 1 \times 10^{19} \text{ m}^{-3}$, $T \sim 5-10 \text{ eV}$
  – $B_{\text{axial}}=0.05 \text{ T}$, $I_{\text{axial}}=5 \text{ kA}$ (at end of ramp)
• Based on spheromak results, temperature-dependent resistivity is expected to be important, so the combined MHD/collisional energy transport modeling is used.
  – $\eta/\mu_0=411 \ T^{3/2} \ \text{m}^2/\text{s} \ (T \ \text{in eV})$, $\chi_\parallel=387 \ T^{5/2} \ \text{m}^2/\text{s}$
• An artificial $D=100 \ \text{m}^2/\text{s}$ and kinematic viscosity of 100 $\text{m}^2/\text{s}$ are used in the computation reported here.
The simulation shows the growth of the \( n=1 \) mode to be faster than exponential after it becomes unstable (while pinching continues).

Magnetic fluctuation energy evolution. Parallel current profile at 0.63 ms.

- The plasma-gun array is modeled as a symmetric heat and particle source at the bottom of the chamber, on axis.
- The simulation shows instability at 0.63 ms, \( I_z=1.3 \) kA.
- The current channel has \( 2\pi rB_z/LB_\phi \approx 1.4 \) at this point.
The fluctuation amplitude is small at initial saturation, but there is appreciable deflection of the current channel with the large aspect ratio.

- There is also a corresponding deflection of the temperature profile (not shown).
- This suggests that magnetic reconnection has occurred, since the location of the temperature source has not changed.

The $\lambda=10$ m$^{-1}$ isosurface shows a deflection of approximately 3 cm over the 1 m path.
Current Filament Modeling

• Electrostatic current drive through biased probes and miniature plasma guns has been accomplished on CDX, CCT [Darrow, et al., PFB 2, 1415 (1990)], and MST.

• Plasma guns also have potential as a means for non-inductive startup in spherical tori and are now being tested on Pegasus [See poster by Eidietis et al.].

• NIMROD simulations are being applied to study the filamentary current channels produced by local sources in vacuum magnetic fields with a large toroidal component.

• Based on spheromak results, temperature-dependent resistivity is expected to be important, so the combined MHD/collisional energy transport modeling is used.
The first set of current filament simulations model the injection tests on Pegasus.

- The mesh represents a 1 m radius chamber with a 5 cm radius center stack.

- Bicubic elements are packed along the current path.

- A finite Fourier series with $0 \leq n \leq 21$ represents the toroidal direction.

- The simulation has the chamber filled with cold but ionized hydrogen at $n=10^{18}$ m$^{-3}$.

- A potential distribution on the bottom surface, similar to the surface temperature distribution, is specified through tangential electric field.

- A local ‘hot spot’ of 2 eV imposed as a boundary condition on the bottom surface represents a miniature plasma gun [Fiksel] source.

- Cooling along the bottom surface outside a thin shell containing the hot spot helps maintain a distinct current channel.
Anisotropic thermal conduction and Ohmic heating produce a spiraling current filament that crosses the chamber axially.

The 2 eV isosurface extends along the magnetic field from the hot spot on the lower boundary to the top surface.

The $J_{||}/B$ isosurface at $-0.2 \text{ m}^{-1}$ (violet) extends along the warm plasma spiral. The applied surface electric field also produces a small amount of return current shown at $+0.2 \text{ m}^{-1}$ (red).
Summary

• The nonlinear evolution of line-tied kink modes is relevant for many laboratory and natural plasma configurations.
• Energy minimization is not suitable for the driven-damped system.
• A new application of minimum-dissipation allows for external control of $\lambda_{\text{inj}}$. The region surrounding the symmetric bias flux is predicted to saturate at $\lambda=2/a$.
• Simulations with a prescribed $<\lambda_{\text{inj}}>$ profile avoid short-circuit current but transition to limit-cycle behavior at low levels of flux amplification.
• An initial 3D simulation of the kink mode in the RWM shows current-path deflection that is comparable to the current channel size.
• The study of helical current injection in Pegasus is of practical interest and is closely related to the general relaxation study.