Effects of Energetic Beam Ions on Stability Properties of Field Reversed Configurations

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OUTLINE:

I. Motivation

- Current drive and stability.

II. Model

- Self-consistent equilibria, full-orbit description for thermal and beam ions.

III. Linear theory

- Derivation of beam-plasma interaction term has been generalized.
- Qualitatively predicts the effects of NBI on stability of MHD modes.

IV. Linear/nonlinear simulation results

- Numerical study of beam ion effects of low-n modes.

V. New regime of stability: E~1

- New stability regime has been found numerically, which requires conducting shell and NBI stabilization.

Stability and current drive are major challenges for FRCs

Motivation for this study:

- New formation methods have been developed.

  Counter-helicity spheromak merging method allows to form FRCs with elongation E~1 and relatively large $S^*=R_s/\rho_i$ ($S^*\sim 20\text{-}30$) [SSX-FRC, MRX-FRC, U. Tokyo].

  For low-n MHD modes, this is MHD-like regime, because for $S^*/E>10$ thermal ion FLR effects are weak.

- External modes can be stabilized by close-fitting conducting shells or shaping:

  MRX-FRC results - the n=1 modes can be stabilized in high mirror-ratio configurations [S. Gerhardt talk, this meeting].

- Stabilization of low-n internal modes requires additional stabilizing mechanisms.

  $n\geq 2$ axial modes in MRX-FRC.

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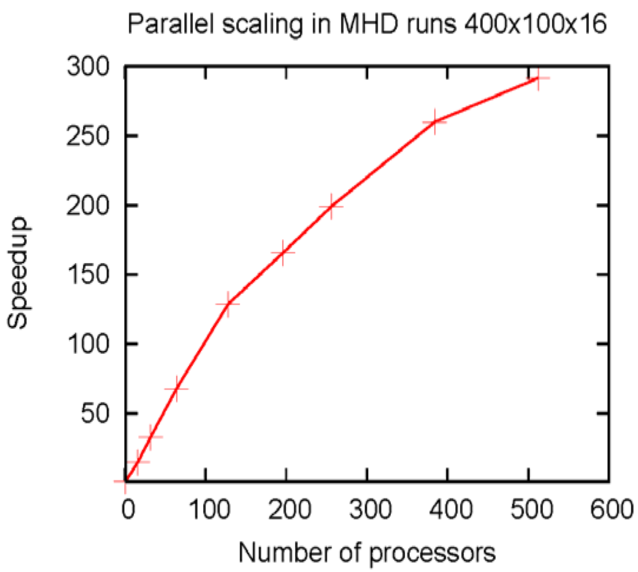
HYM – Parallel Hybrid/MHD Code

HYM code developed at PPPL and used to investigate FRC formation and stability properties

- 3-D nonlinear.
- Three different physical models:
  - Resistive MHD & Hall-MHD.
  - Hybrid (fluid electrons, particle ions).
  - MHD/particle (one fluid thermal plasma, + energetic particle ions)
- Full-orbit kinetic ions.
- For particles: delta-f / full-f numerical scheme.
- Parallel (3D domain decomposition, MPI)[1].
- Self-consistent equilibria, including beam ion effects.



N_{procs}

New MPI version of HYM shows good parallel scaling up to 500 processors for production-size jobs, and allows high-resolution nonlinear simulations.

[1]Simulations are performed at NERSC.

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Oblate FRCs have different stability properties than prolate FRCs

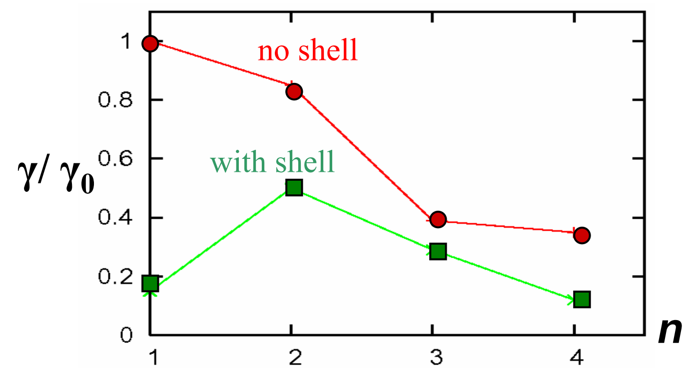
Prolate FRCs (E >> 1)

- All unstable modes (n≥1) are internal.
- Conducting shell has little effect on stability.
- FLR effects are stabilizing for low *S*/E*.

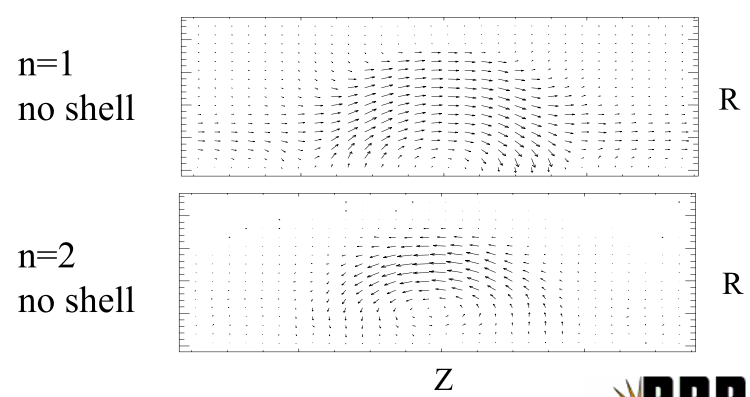
Oblate FRCs (E<1)

- n=1 *tilt and radial shift* modes can be stabilized by conducting shells/shaping.
- *Interchange modes* (n≥1) can be stabilized by profile effects.
- n>1 *co-interchange (kink-like)* modes are internal modes, and they remain unstable in the presence of a close-fitting conducting shell.
- S*/ E >>1 → FLR stabilization is weak.

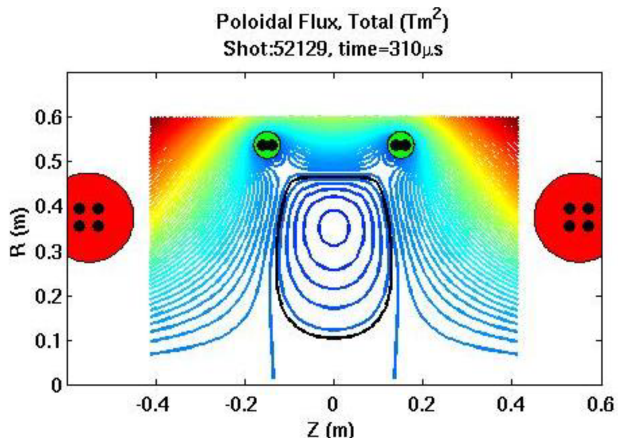
Hybrid simulations E=1.1



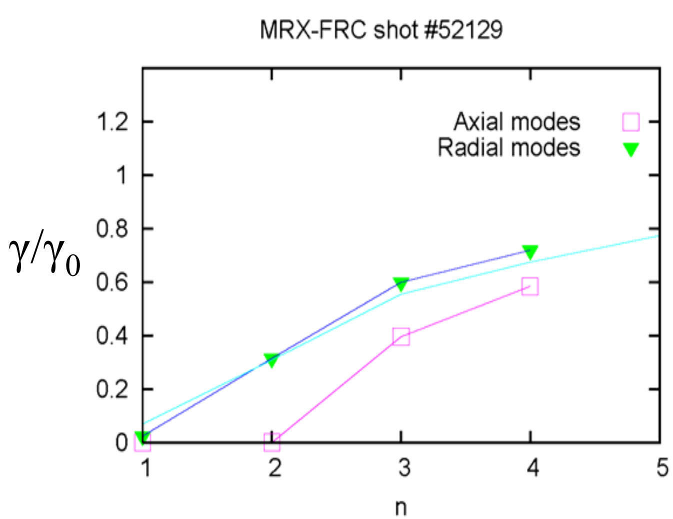
Normalized growth rates of the *n*=1-4 modes from 3D hybrid simulations including kinetic effects (red) and the effects of conducting shell (green) for E=1.1 and flat current profile.



MRX FRC experiments show improved stability for large mirror ratio, small elongation (E<0.5) cases.



MRX-FRC equilibrium [S. Gerhardt]



Normalized linear growth rate vs toroidal mode number from MHD simulations using HYM code.

- Two types of co-interchange modes are observed:
  - radially-polarized modes,
  - axially-polarized modes.

  [For n=1 these are the radial shift mode and tilt mode, respectively.]

- Strong equilibrium field shaping (large mirror ratio) can stabilize the n=1 modes, and improve stability of n=2 modes.

- Effects of external field shaping are much weaker for higher-n modes (n≥3) due to more localized structure of these modes.

- Energetic beam ions can provide additional stabilizing mechanism for n≥3 modes.

Injection of energetic ion beams may provide additional stabilizing mechanism, as well as plasma heating and current drive

Previous studies (equilibrium and stability)

- Self-consistent equilibrium calculations show that the beam ions tend to coalesce between the magnetic null and the separatrix near the midplane, and an increase in the beam current reduces the axial spreading of the beam [Barnes`91, Nishimura`99, Lifschitz`02].

- Generalized energy principle [Lovelace'75, Finn and Sudan'93] shows that the bulk plasma-beam interaction term is proportional to

$$\mathrm{I_b} \sim 1/[n^2\Omega^2 - \omega_\beta{}^2],$$

  where Ω is average toroidal rotation frequency of the beam ions, and ω_β is the betatron frequency.
  The beam ion contribution is stabilizing provided **$n|\Omega| > \omega_\beta$**.

- 3D numerical simulations [Barnes'91] found the stabilization of the tilt instability for prolate FRC with n_b~1-2% and V_0~$10V_A$; destabilization of the *n*=4 kink mode has been found for cold beams [Nishimura`99].

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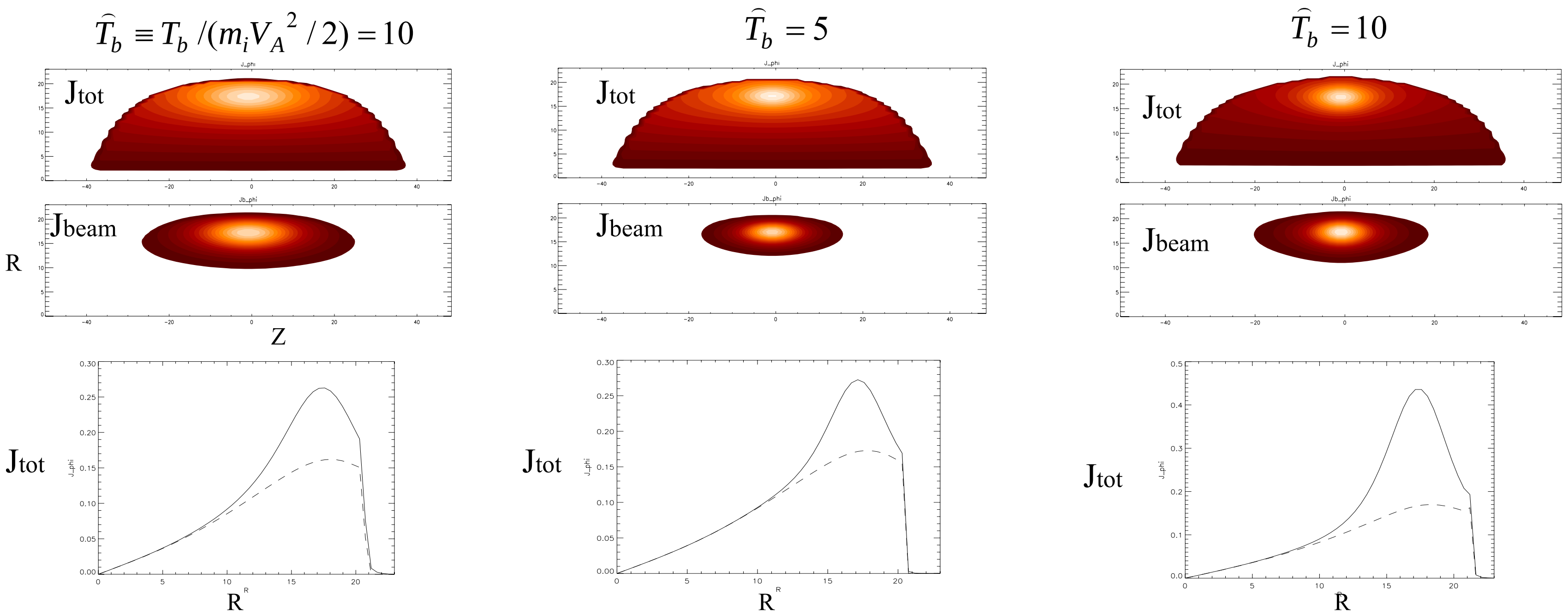
FRC equilibrium properties including the effects of NBI ions

- Beam ions tend to coalesce between the magnetic null and the separatrix near the FRC midplane. The beam is strongly localized when the beam ion temperature is small ($V_{th,b} << V_0$) or when beam current is large ($J_b \sim J_{tot}$). Larger beam ion temperature or weaker beams result in a broader beam profiles.



Contour plots of the toroidal total and beam ion current, and radial profiles of the total and bulk plasma current density for FRC with E=1.7, $V_0=6.2V_A$, and different values of normalized beam ion temperature and $n_b=0.015$ and $n_b=0.04$.

Linear stability: Beam – plasma interaction term

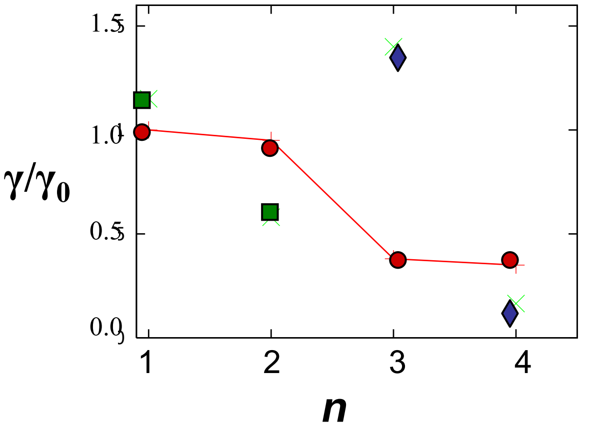
Thermal plasma-beam interaction term (assuming $\Omega \gg \omega$) can be shown to be for odd modes (antisymmetric relative to the midplane, ie with tilt-like polarization):

$$I_{\text{odd}} = \frac{A_1}{n^2\Omega^2 - \omega_\beta^{\,2}} + \frac{A_3}{n^2\Omega^2 - 9\omega_\beta^{\,2}} + \ldots$$

For even modes (symmetric, with radial polarization):

$$I_{\text{even}} = \frac{A_2}{n^2\Omega^2 - 4\omega_\beta^{\,2}} + \frac{A_4}{n^2\Omega^2 - 16\omega_\beta^{\,2}} + \ldots$$

where Ω is the toroidal rotation frequency, and ω_β is the betatron frequency of the beam ions.



Growth rates of low-n MHD modes without NBI (red), and including the energetic beam ion contribution (green and blue) for FRC equilibrium with E=1.1 and beam parameters: n_b =3%, Ω_0/ω_{ci} =0.4, T_b =10. Only the most unstable modes are shown for each *n*.

For FRC equilibrium with E~1 and the beam with average frequencies $\langle\Omega/\omega_{ci}\rangle$ =0.4 and $\langle\omega_\beta/\omega_{ci}\rangle$ =0.63, the beam effects are:

- n=1 odd – destabilized, even – weakly stabilized;
- n=2 odd – partially stabilized, even – weakly destabilized;
- n=3 odd – weakly stabilized, even – strongly destabilized.

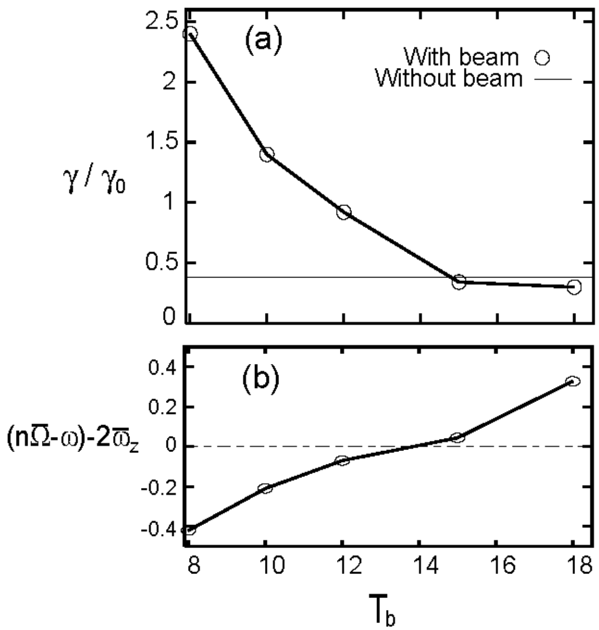
Linear stability

- The beam ion contribution is stabilizing provided ***I* > 0**. For the *n*=1 mode, this condition can be satisfied only for weak beams, i.e. with small beam ion current density.

- Strong beams typically have $\Omega<\omega_\beta$ (i.e. can be destabilizing).

- Self-consistent inclusion of the beam ion effects results in the modification of equilibrium profiles: an increase in the separatrix radius, reduction of E, and more peaked current profiles.

- Calculations have been performed for E~1-2, n_b/n_i=0.01-0.05, and V_0=4-6V_A.

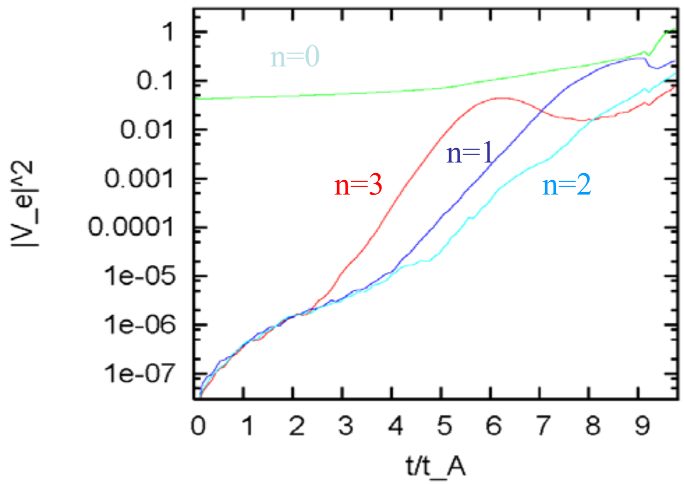


Growth rates of the n=3 mode as function of beam ion temperature. Correlation with sign of $[(3\Omega-\omega) -2\,\omega_\beta]$.

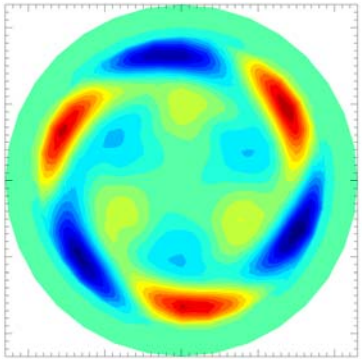
Nonlinear simulation results

The beam-destabilized *n*=3 mode saturates nonlinearly at small amplitude, the *n*=1 tilt mode becomes the most unstable mode in the nonlinear regime.

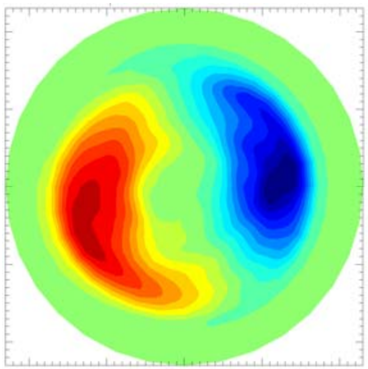
$E=1.1$, $n_b=3\%$, $\Omega_0/\omega_{ci}=0.4$, $T_b=10$.



Time evolution of amplitudes of n=0-3 Fourier harmonics from 3D nonlinear hybrid simulations including the beam ions. The n=3 mode is the most unstable linear mode, but it saturates nonlinearly; the n=1 tilt mode grows to large amplitudes until it destroys the configuration.



$t=5.2\ t_A$
$\delta n= 0.1$



$t=8.7\ t_A$
$\delta n= 0.3$

Contour plots of the density perturbation at the FRC toroidal cross-sections.

Agree with previous results: nonlinear stabilization of the *n*=4 kink mode for cold beams [Nishimura`99]; and nonlinear simulations by [Barnes’91].

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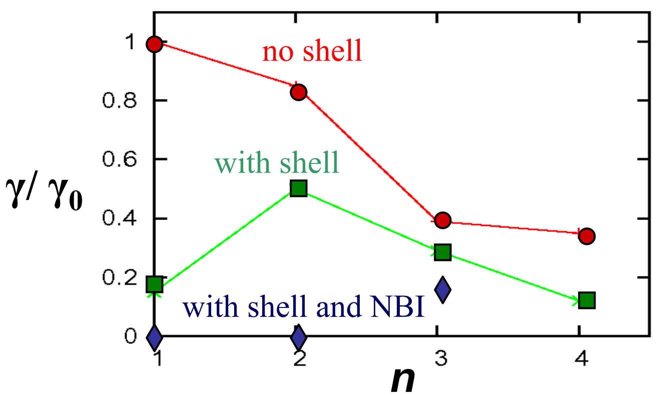
FRC stability including close-fitting conducting shell and energetic beam ion effects: I. Linear results

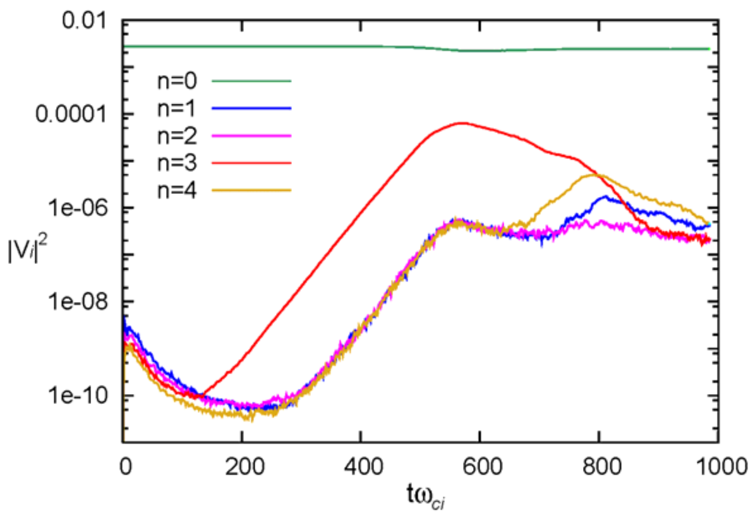
- Close-fitting conducting shell stabilizes all low-*n* radially-polarized (even) modes.
- Due to localization, the ion beams are effective in stabilizing the residual low-*n* instabilities, except for relatively cold beams which have a destabilizing effect on $n \geq 3$ modes.
- The NBI effects are stronger for lower-*n* modes (*n*=1 and *n*=2), and smaller V_0.
- The n=1 tilt mode and the n=2 mode are stabilized, and the growth rate of the n=3 mode is reduced for E~1, n_b/n_i=0.03, and V_0= $6V_A$.

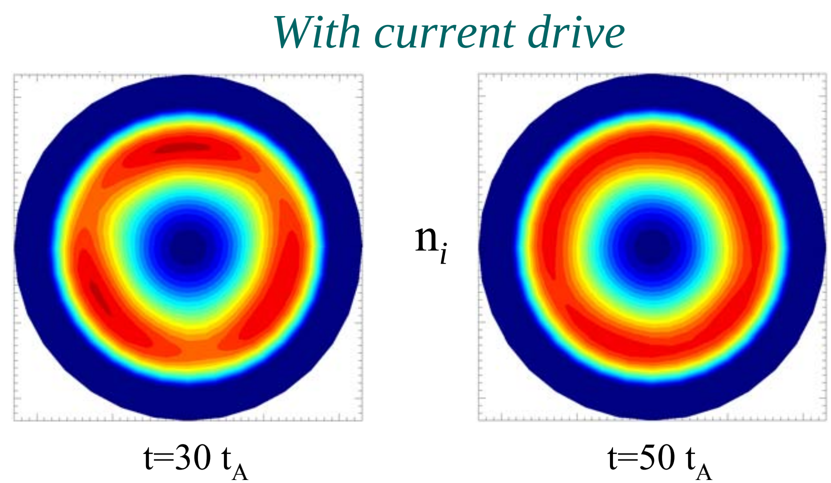


Normalized growth rates of the *n*=1-4 modes from 3D hybrid simulations including the effects of conducting shell and NBI stabilization.

FRC stability including close-fitting conducting shell and energetic beam ion effects: II. Nonlinear simulations

- Nonlinear 3D simulations show that the residual instabilities (n=3 mode) saturate at small amplitudes.
- FRC remains stable with respect to **all MHD modes**, as long as it is sustained.





Nonlinear hybrid simulations of an FRC with E=1.1, including the effects of the beam ions and the close-fitting conducting shell. (a) Time evolution of n=0-4 modes kinetic energy; and (b) contour plots of plasma density in the toroidal cross sections.

Simulations have been performed in support of MRX-FRC experimental proposal.

Conclusions

- The energetic beam ions tend to coalesce between the magnetic null and the separatrix near the FRC midplane. For strong beams, the beam profiles are highly localized.
- Due to localization, the ion beams can significantly modify stability properties of low-n MHD modes.
- Effects of the beam ions on the MHD stability depend on the MHD mode polarization, and beam ion parameters: the average toroidal and betatron frequencies of the beam.
- Beam-driven instabilities saturate nonlinearly.
- A new stability regime has been discovered for oblate FRCs with a close-fitting conducting shell and energetic beam ion stabilization.
  - *Linearly stable with respect to the n=1 tilt mode and the n=2 modes,*
  - *Residual instabilities saturate nonlinearly at small amplitudes,*
  - *Configuration remains MHD stable, if current is sustained.*

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